

IUT 1st Admission Test(SBL)

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- ☐ The points for each question are listed next to the question number.
- ☐ You can use the right side of each page for your memo.

1. [3 points]

Find $(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4$.

- ① 30 ② 32 ③ 34 ④ 36 ⑤ 38

2. [3 points]

When $a + b + c = 1$, $a^2 + b^2 + c^2 = 5$ and

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 3$, find abc .

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 0 ④ $-\frac{2}{3}$ ⑤ $-\frac{4}{3}$

3. [3 points]

When α and β are the solutions of $x^2 + 5x + 2 = 0$, find $\alpha^3 + \alpha\beta + \beta^3$.

- ① -91 ② -93 ③ -95 ④ -97 ⑤ -99

4. [3 points]

When $t - \frac{1}{t} = 2\sqrt{3}$ and $t > 0$, find $t^3 + \frac{1}{t^3}$.

- ① 50 ② 52 ③ 54 ④ 56 ⑤ 58

5. [3 points]

Find $\sum_{n=1}^{10} \frac{1}{n^2 + 4n + 3}$.

- ① $\frac{31}{104}$ ② $\frac{33}{104}$ ③ $\frac{35}{104}$ ④ $\frac{37}{104}$ ⑤ $\frac{39}{104}$

6. [3 points]

When (a_1, b_1) and (a_2, b_2) are solutions of

$$\begin{cases} x^2 + y^2 = 2 \\ 2x + y = 1 \end{cases},$$

find $a_1 + b_1 + a_2 + b_2$.

- ① $-\frac{4}{5}$ ② $-\frac{2}{5}$ ③ 0 ④ $\frac{3}{5}$ ⑤ $\frac{6}{5}$

7. [3 points]

Simplify

$\log_3(\sqrt{2} + \sqrt{8} + \sqrt{9}) + \log_3(\sqrt{2} + \sqrt{8} - \sqrt{9})$.

- ① 2 ② 3 ③ $\sqrt{2}$ ④ $\sqrt{3}$ ⑤ $\sqrt{2} + \sqrt{3}$

8. [3 points]

Find the sum of all solutions of

$$4^x - 5 \cdot 2^x + 2 = -8 \cdot 2^{-x}.$$

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

9. [3 points]

Simplify $\left(3^{\log_{\sqrt{3}} 8}\right) - \left(\frac{1}{2}\right)^{\log_4 \left(\frac{1}{81}\right)}$.

- ① 51 ② 53 ③ 55 ④ 57 ⑤ 59

10. [3 points]

When $2^x = \frac{1}{\sqrt{3}}$ and $4^y = 27$, find $\frac{y}{x}$.

- ① -1 ② -3 ③ -5 ④ -7 ⑤ -9

11. [3 points]

When $a = \sqrt{3} + i$ and $b = \sqrt{3} - i$, find $\frac{a^3 - b^3}{ab}$.

- ① $2i$ ② $4i$ ③ $6i$ ④ $8i$ ⑤ $10i$

12. [3 points]

Compute $\operatorname{tg} \frac{\pi}{8}$, where $\operatorname{tg} \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $-1 + \sqrt{2}$ ② $-\frac{1}{2} + \sqrt{2}$ ③ $-\frac{1}{\sqrt{2}} + \sqrt{2}$
④ $-\frac{1}{3} + \sqrt{2}$ ⑤ $-\frac{1}{\sqrt{3}} + \sqrt{2}$

13. [3 points]

Find the sum of all solutions of

$$3 \cos 2x + 2 \cos x - 1 = 0, \quad (0 \leq x \leq 2\pi).$$

- ① π ② $\frac{3}{2}\pi$ ③ 2π ④ $\frac{5}{2}\pi$ ⑤ 3π

14. [3 points]

Find the sum of all solutions of

$$\sqrt{3} \sin x - \cos x = \sqrt{3}, \quad (0 \leq x \leq 2\pi).$$

- ① $\frac{\pi}{2}$ ② $\frac{2\pi}{3}$ ③ $\frac{5\pi}{6}$ ④ π ⑤ $\frac{4\pi}{3}$

15. [3 points]

When $A = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$, and

$A^{-1}B^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a + b + c + d$.

- ① -6 ② -3 ③ 0 ④ 3 ⑤ 6

16. [3 points]

When $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $A^{100} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,
find $a + b + c + d$.

- ① 202 ② 204 ③ 206 ④ 208 ⑤ 210

17. [3 points]

Find $\lim_{x \rightarrow 0} \frac{x^3 + 2 \sin x (1 - \cos x)}{x (1 - \cos x)}$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

18. [3 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{x^3 + 3} - \sqrt{x^3 + 3x\sqrt{x} + 4})$.

- ① $-\frac{1}{2}$ ② -1 ③ $-\frac{3}{2}$ ④ -2 ⑤ $-\frac{5}{2}$

19. [3 points]

When $y = ax + b$ is the tangent line to
 $f(x) = \frac{\sqrt{x+4}}{2x+1}$ at $x = 0$, find $a + b$.

- ① $-\frac{1}{4}$ ② $-\frac{3}{4}$ ③ $-\frac{5}{4}$ ④ $-\frac{7}{4}$ ⑤ $-\frac{9}{4}$

20. [3 points]

Let M and m be the maximum and minimum
values of $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 1$,
($-3 \leq x \leq 3$), respectively. Find $M + m$.

- ① $\frac{20}{3}$ ② $\frac{22}{3}$ ③ 8 ④ $\frac{26}{3}$ ⑤ $\frac{28}{3}$

21. [4 points]

Find the minimum value of

$$f(x) = 3\sin^2 x - 4\sin x \cos x + 2.$$

- ① -2 ② -1 ③ 0 ④ 1 ⑤ 2

22. [4 points]

When $\omega = \frac{1 - \sqrt{3}i}{2}$, find $\sum_{n=0}^{14} \omega^n$.

- ① 0 ② 1 ③ $\sqrt{3}i$
④ $1 - \sqrt{3}i$ ⑤ $\frac{1 - \sqrt{3}i}{2}$

23. [4 points]

When $f(x) = \sqrt[3]{(3x+2)^4 - 8}$, find $f'(0)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

24. [4 points]

When a continuous function $f: [0, \infty) \rightarrow \mathbb{R}$ satisfies $\int_0^x f(t^2) dt = x \sqrt{2x^2 + 1}$, find $f(4)$.

- ① $\frac{11}{3}$ ② $\frac{13}{3}$ ③ $\frac{14}{3}$ ④ $\frac{16}{3}$ ⑤ $\frac{17}{3}$

25. [4 points]

Find the minimum value of $f(x) = x^4 + 2x^3 + 4x^2 - 6x + 2$.

- ① $\frac{1}{16}$ ② $\frac{3}{16}$ ③ $\frac{5}{16}$ ④ $\frac{7}{16}$ ⑤ $\frac{9}{16}$

26. [4 points]

Find $\int_1^4 \frac{x^2 - 2}{\sqrt{x}} dx$.

- ① $\frac{42}{5}$ ② $\frac{44}{5}$ ③ $\frac{46}{5}$ ④ $\frac{48}{5}$ ⑤ 10

27. [4 points]

Find $\int_0^1 \frac{1}{(2x+1)^3} dx$.

- ① $\frac{2}{9}$ ② $\frac{4}{9}$ ③ $\frac{2}{3}$ ④ $\frac{8}{9}$ ⑤ $\frac{10}{9}$

28. [4 points]

Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2 + 2kn - n^2}{n^3}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

29. [4 points]

When $\int_0^1 f(2x) dx = 3$ and $\int_0^3 f(x) dx = 18$, find $\int_2^3 f(x) dx$.

- ① 12 ② 14 ③ 16 ④ 18 ⑤ 20

30. [4 points]

Find the area of the region enclosed by $y = 3x^3 + 2x^2 + x + 5$ and $y = 3x^3 + x^2 + 4x + 3$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{6}$