ARADA SUB-CITY EDUCATION BUREAU GRADE 12 MATHEMATICS SECOND SEMESTER MODEL EXAM ONE, APRIL 2024

NUMBER OF QUESTIONS: 65 **TIME ALLOWED:- 3 HOURS**

GENERAL DIRECTIONS

THIS EXAM IS **MATHEMATICS FOR NATURAL SCIENCE STUDENTS ONLY**.

IN THIS EXAMINATION, THERE ARE A TOTAL OF 65 **MULTIPLE CHOICE QUESTIONS**. CAREFULLY SELECT THE BEST ANSWER AND **BLACKEN** ONLY THE LETTER OF YOUR CHOICE ON THE SEPARATE ANSWER SHEET PROVIDED. FOLLOW THE INSTRUCTIONS ON THE ANSWER SHEET AND THE EXAMINATION PAPER CAREFULLY. USE ONLY **PENCIL** TO MARK YOUR ANSWERS. YOUR ANSWER MARK SHOULD BE **HEAVY** AND **DARK**, COVERING THE ANSWER SPACE COMPLETELY. PLEASE ERASE ALL UNNECESSARY MARKS COMPLETELY FROM YOUR ANSWER SHEET.

YOU ARE ALLOWED TO WORK ON THE EXAM FOR **3 HOURS**. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, PUT YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMPT TO CHEAT IN THE EXAMINATION WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE (S).

PLEASE MAKE SURE THAT YOU HAVE WRITTEN ALL THE REQUIRED INFORMATION ON THE ANSWER SHEET BEFORE YOU START TO WORK ON THE EXAMINATION.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO'

DIRECTION: Each of the following questions is followed by four possible alternatives. Read each question carefully and BLACKEN the letter of your choice on the answer sheet provided.

1. If x and y are composite natural numbers and z is a prime number, then which one of the following is **never** true? A. $x + y = z$ B. $x + y = 2z$ C. $xz = y$ D. $xy = z^2$

2. The simplified form of the expression $\left(\frac{5}{4}\right)^3 \left(\frac{\sqrt{5}}{2}\right)^{-4} \div \left(\frac{\sqrt[3]{5}}{\sqrt{2}}\right)^6$ is:

   A. $\frac{5}{2}$ B. $\frac{2}{5}$ C. $\frac{1}{5}$ D. 5

3. To rationalize the denominator $\frac{3+\sqrt{2}}{4-\sqrt{3}}$ which one of the following expression is used as the rationalizing factor? A. $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ B. $\frac{3-\sqrt{2}}{4+\sqrt{3}}$ C. $\frac{4+\sqrt{3}}{4+\sqrt{3}}$ D. $\frac{1}{4+\sqrt{3}}$

4. *What is the value of* x *for* $2\left(\frac{243}{32}\right)^{2x} = 3\left(\frac{8}{27}\right)^{\frac{2}{3}x-1}$*?* *A.* $\frac{1}{3}$ *B.* $\frac{4}{3}$ *C.* $\frac{2}{5}$ *D.* $\frac{1}{4}$

5. Which one of the following systems of linear equation has infinitely many solutions?

   A. $\begin{cases} 3x + y = -1 \\ -x - \frac{1}{3}y = \frac{1}{3} \end{cases}$ B. $\begin{cases} x + y = 6 \\ 2x - 5y = 0 \end{cases}$ C. $\begin{cases} x + 3y = 2 \\ 2x - 4y = 1 \end{cases}$ D. $\begin{cases} x - 2y = 5 \\ -3x + 6y = 1 \end{cases}$

6. What is the solution set of the inequality $|3x + 6| - 2 \geq 4$?

   A. $(-\infty, -4) \cup (0, \infty)$ B. $[-4,0]$ C. $(-\infty, -4] \cup [0, \infty)$ D. $(-4,0)$

7. The solution set of the inequality $3x^2 + 2x - 1 < 0$ is:

   A. $(-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$ B. $\left(-1, \frac{1}{3}\right)$ C. $\emptyset$ D. $x: x \in \mathbb{R}$

8. For any two non-empty sets A and B. If $A \subseteq B$, then which one of the following is always **true**?

   A. $A \cap B = B$ B. $A \cup B = A$ C. $A \cap B = A$ D. $(A \cup B) \cap A = B$

9. Let $\Re = \{(x, y): y < 5 \text{ and } y \geq x^2 - 4\}$. Then, which one of the following is **true** about the relation $\Re$?

   A. Domain of $\Re$ is $[-3,3]$
   B. Range of $\Re$ is $[-4,5]$
   C. Domain of $\Re$ is $[-3,3)$
   D. Range of $\Re$ is $[-4,5)$

10. Which one of the following is **Not** true about the graph of the function $f(x) = 2x^2 - 8x + 3$?

    A. It's range is $y: y \geq -5$.
    B. It's vertex is $(2, -5)$.
    C. It's axis of symmetry is $x = 2$.
    D. It's maximum value is -5.

11. If $f(x) = \frac{2x+4}{x-1}$. Then, which one of the following points is on the graph of $f^{-1}(x)$?

A. $(1,-5)$ B. $(3,5)$ C. $(0,-4)$ D. $(2,8)$

12. If the ratio of the areas of two similar triangls are $64cm^2$ *and* $81cm^2$, then which one of the following is **Not** true?

A. The ratio of their perimeter is $8:9$.
B. The ratio of their sides is $8:9$.
C. The ratio of their perimetr is $64:81$.
D. All the ratio of their corresponding sides are equal.

13. If ΔABC a right angle triangle right at C such that $\sin A = \frac{5}{13}$, then which one of the following is **Not** true?

A. $\cos A = \frac{12}{13}$
B. $\tan A = \frac{5}{12}$
C. The area of ΔABC is $30sq\ units$
D. The ratio of $\overline{BC}\ to\ \overline{AC}$ is necessarily $\frac{12}{5}$.

14. If the angle of a sector at the center of a circle is 180^0 with radius $4cm$. Then, what is the area of the sector? A. $8\pi cm^2$ B. $4\pi cm^2$ C. $16\pi cm^2$ D. $2\pi cm^2$

15. Which one of the following is **Not** true about a parallelogram ?

A. Parallelogram is a quadrilateral with opposite sides parallel.
B. The consecutive angles of a parallelogram are supplementary.
C. The diagonals of a parallelogram are equal.
D. The opposite angles of a parallelogram are congruent.

16. If a regular $8-sided$ polygon is inscribed in a circle of radius $2cm$, then what is the area of the polygon in cm^2? A. 32 B. $32\sqrt{2}$ C. 8 D. $8\sqrt{2}$

17. The base of a regular pyramid is an equilateral triangle of side $8cm$ long. If the altitude of the pyramid is $9cm$, then what is the volume of the pyramid ?

A. $16\ cm^3$ B. $48cm^3$ *C.* $16\sqrt{3}cm^3$ *D.* $48\sqrt{3}\ cm^3$

18. Let the radii of bases of a frustum of cone are $6cm\ and\ 3cm$. If the slant height of a frustum of a cone is $10\ cm$, then what is the total surface area of the frustum?

A. $90\pi\ cm^2$ B. $45\pi cm^2$ *C.* $135\pi cm^2$ *D.* $180\pi\ cm^2$

19. Suppose the mid-point of a line segment is $M(-2,3)$. If one end point of the segment is $P(4,-8)$, then what is the coordinate of the other end-point?

A. $(-4,6)$ *B.* $(3,-5)$ *C.* $(-8,14)$ *D.* $(6,3)$

20. What is the area of the triangle in $(sq.unit)$ formed by the line joining the vertex of the parabola $x^2 = 64y$ to the end points of the latus rectum?

A. 64 *B. 128* *C. 256* *D. 512*

21. *The equation of an ellipse with center at* $(1,5)$ *and vertices at* $(10,5)$ *and* $(1,2)$ *is:*

A. $(x-1)^2+9(y-5)^2=1$ *C.* $9(x-1)^2+(y-5)^2=1$

B. $(x-1)^2+9(y-5)^2=81$ *D.* $9(x-1)^2+(y-5)^2=81$

22. *If the standard deviation of* $x_1, x_2, x_3, \ldots x_n$ *is* 6 *. Then , what is the variance of*

$2x_1+4,\ 2x_2+4,\ 2x_3+4,\ 2x_1+4$ *A. 16 B. 144 C. 76 D. 148*

23. *The following is the distribution of the weight of 36 persons.*

Weight in kg	*40 - 49*	*50 -59*	*60 - 69*	*70 -79*
Frequency	*6*	*10*	*17*	*3*

Then, what is the 20^{th} *percentile* (p_{20}) *of the measurement?*

A. 50.7 B. 50.4 C. 51.4 D. 51.7

24. *Let a sample space be* $s=\{w_1,w_2,w_3,w_4\}$*. Which of the following assignment of probabilities to each outcome are valid?*

Assignment	w_1	w_2	w_3	w_4
P	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$
Q	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
R	0	1	0	0
S	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0
T	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{4}$	$\frac{1}{3}$

A. Q, R *and* T *B.* Q, R *and* S *C.* Q, S *and* T *D.* P, S *and* T

25. *Given the data*

x	*2*	*3*	*4*	*6*
f	*5*	*4*	*2*	*1*

Then, what is the mean deviation about the mode of the data set?

A. 1 *B.* $\frac{1}{4}$ *C.* $\frac{1}{2}$ *D.* $\frac{1}{6}$

26. *The mean of 100 observation was calculated as 40 by a student who took mistake 40 instead of 90 for one observation. Then, what is the correct mean? A. 39.5 B. 40.5 C. 41 D. 41.5*

27. *If the mean of the data* $6,3,2,9,x,9$ *is 5, then what is the mean deviation about the mean?*

A. 4 B. 1 C. 2 D. 3

28. *The coefficient of mean deviation about the median of the data set*

x	*1*	*2*	*3*	*4*
f	*2*	*4*	*3*	*1*

is: A. 0.7 B. 0.35 C. 0.6 D. 0.45

29. *The median and the mean deviation about the median of gross incomes of four companies are given below:*

Company	*md(median)*	*MD(md)*
W	*40,000*	*7,000*
X	*30,000*	*6,000*
Y	*50,000*	*4,000*
Z	*60,000*	*8,000*

Then, which company income is most consistent? A. W B. X C. Y D. Z

30. *Which one of the following is always **true**?*

A. *If the mode, median and mean of a distribution are 4,5 and 6 respectively, then the distribution is negatively skewed.*

B. *If the $1^{st}, 2^{nd}$ and 3^{rd} quartiles of a distribution are 7,9 and 12 respectively, then the distribution is symmetrical.*

C. *If the mean, median and standard deviation of a distribution are 4,6 and 2 respectively, then the distribution is skewed to the right.*

D. *If the distribution is negatively skewed, then the mean of the distribution is less than its second quartile.*

31. *In Addis Ababa there are 100 secondary school with 100,000 grade 12 students in total. Suppose Addis Ababa Education Bureau has selected 1,000 students from them for survey purposes by collected students' roster from all the schools and combined them into one. After assigning a new roll number to each students, the Bureau selected those numbers of students* 40,140,240,340,440, … *, then which sampling method is the Bureau used?*

A. *Simple random* B. *Systematic* C. *Cluster* D. *Stratified*

32. *Let ABCD is a parallelogram and $\overline{AC}$ and $\overline{BD}$ be diagonals, then which one of the following is **Not** true?*

A. $\overline{AB} + \overline{BC} = \overline{AC}$ C. $\overline{DC} + \overline{AD} = \overline{AC}$

B. $\overline{AB} = \overline{BC} - \overline{BD}$ D. $\overline{AB} - \overline{BC} = \overline{DC}$

33. When $x^3 + 3x^2 - kx + 5$ is divided by $x + 2$, the remainder is 7. Then, what is the value of k?

A. 1 B. -2 C. -1 D. 2

34. Which one of the following is **true** about polynomial function ?

A. The domain of polynomial function is the set of positive real number

B. The graph of a polynomial function of degree n-has at most n-turning points.

C. The graph of every polynomial function crosses the y-axs.

D. The graph of a polynomial function of degree n-has at most n-1 zeros.

35. *Let $f(x) = \left(\frac{3}{2}\right)^x$ and $g(x) = \left(\frac{2}{3}\right)^x$. Then, which one of the following is **Not** true about $f(x)$ and $g(x)$?*

A. *If $> 0, then\ f(x) < g(x)$.* C. *If $x < 0, then\ f(x) < g(x)$.*

B. *if $x = 0, then\ f(x) = g(x)$.* D. *If $x > 0, then\ f(x) > g(x)$.*

36. *Which of the following is the solution set of the equation* $\log_3 x + \log_{\sqrt{x}} 4x + \log_{\frac{1}{3}} x = 6$*?*

A. $\{2\}$ B. $\{-2\}$ C. $\emptyset$ D. $\{-2, 2\}$

37. If $\tan(\theta - 60°) = \cot 4\theta$. Then, what is the value of θ?

A. $60°$ B. $40°$ C. $50°$ D. $30°$

38. If the angle of elevation of the top of the mountain from the top of the building is 30° and the angle of elevation of the top a mountain from the bottom of a building 105m high is 45° and the distance between the bases of the building is 40m. Then, what is the height of the mountain and the distance between from the top of the building to the top of the mountain respectively?

A. $45m, 15m$ B. $105m, 7.5m$ C. $95m, 15m$ D. $15m, 7.5m$

39. Which one of the following is **Not** true about the graph of $f(x) = \frac{x^3-x}{x^4-x^3}$?

A. The graph has a hole at point $(1,2)$.

B. The graph intersects its horizontal asymptote at point $(-1,0)$.

C. The y-axis is the vertical asymptote of the graph.

D. The graph is symmetric with respect to the origin.

40. For any real number x *and* y which one of the following statement is true?

A. $(\forall_x)(\exists_y)(y^2 + x + 1 = 0)$ C. $(\forall_y)(\exists_x)(y^2 + x + 1 = 0)$

B. $(\exists_x)\forall_y)(y^2 + x + 1 = 0)$ D. $(\exists_y)(\forall_x)(y^2 + x + 1 = 0)$

41. If A is a 3×3 matrix and $|A^{-1}| = 3$, then what is the $|6A^TA|$?

A. 72 B. 36 C. 24 D. 18

42. For *What value(s) of k does* $\begin{cases} x + 2z = 4 \\ 3x + 4y + 4z = 6 \\ kx + 3y - z = 1 \end{cases}$ *has a unique solution?*

A. $k = 4$ *B.* $k = \frac{1}{4}$ *C.* $k \neq 4$ *D.* $k \neq \frac{1}{4}$

43. Let Z be a complex number and $a, b \in \mathbb{R}$ *. Then, which one of the following is* ***Not*** *true?*

A. $z.\bar{z} = |z|^2$ *C. If* $z = -bi - a$, *then* $\bar{z} = bi - a$

B. If $Z = a + bi$, *then* $z + \bar{z} = 2a$ *D. If* $z \neq 0$, *then it is multiplicative inverse is* $\frac{z}{(z)^2}$.

44. If the translation T takes the point (3,1) to the point (4,0). Then, what is the image of the circle $x^2 + y^2 = 2$ under the translation T?

A. $x^2 - 2x + y^2 + 2y = 0$ C. $x^2 + 2x + y^2 + 2y = 0$

B. $x^2 + 2x + y^2 - 2y = 0$ D. $x^2 - 2x + y^2 - 2y = 0$

45. What is the amplitude and period of the graph of $f(x) = -4\sin\left(\frac{1}{4}x\right)\cos\left(\frac{1}{4}x\right)$?

A. $4, \pi$ B. $2, 4\pi$ C. $4, 2\pi$ D. $2, 2\pi$

46. A patrol on a sea sailed from it's station $4\sqrt{2}km$ to the south and changed it's course and sailed 7km in the direction of 45^0 North-west. Then, what is the shortest(straight) distance the boat should travel in order to return to its station?

A. $5\sqrt{2}km$ B. $5km$ C. $(5 + \sqrt{2})km$ D. $7km$

47. *Which one of the following is the fourth term of the sequence* $\left\{\cos(\frac{n\pi}{3})\right\}_{n=0}^{\infty}$*?*

A. 0 *B.* 1 *C.* $-\frac{1}{2}$ *D.* -1

48. *If there are 8 arithmetic means between* 26 *and* -1 *with* 26 *being the first term. Then, what is the common difference?* *A.* -3 *B.* 2 *C.* -2 *D.* 3

49. *If the second and sixth terms of a geometric progression are* 4 *and* $\frac{1}{4}$ *respectively, then what is the sum of the first eight terms 0f the sequence?*

A. $\frac{255}{16}$ *B.* $\frac{127}{8}$ *C.* $\frac{63}{4}$ *D.* $\frac{129}{32}$

50. What is the sum of the of the infinite series $\frac{1}{2}-\frac{2}{5}+\frac{1}{8}-\frac{2}{25}+\frac{1}{32}-\frac{2}{125}+\cdots$?

A. $\frac{7}{10}$ *B.* $\frac{1}{6}$ *C.* $\frac{3}{10}$ *D.* $\frac{7}{6}$

51. A contest offers a total of 10 prizes. The first prize is worth Birr $20{,}000$ and each consecutive prize is worth Birr 1500 less than the next higher prize. What is the total value of the prizes in Birr? A. 135,000 B. 132,500 C. 142,300 D. 141,500

52. A ball is dropped from the height of $16m$ to the horizontal floor. In each bounce it rebounds 60% Of the distance it fell. What maximum possible(total) vertical distance that could be covered by the ball ? (Assume that the ball never comes to rest).

A. $240m$ B. $120m$ *C.* $64m$ D. $80m$

53. Which one of the following series is convergent?

A. $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{-n}$ B. $\sum_{n=1}^{\infty}(-2)^n$ C. $\sum_{n=1}^{\infty}2^{-n}$ D. $\sum_{n=1}^{\infty}(\sqrt{2})^n$

54. What is the absolute maximum value of the function $f(x)=x^3-12x+4\ on\ [-3,3]$?

A. 20 B. 13 C. 24 D. 18

55. In which interval that the graph of the function $f(x)=\frac{x}{x^2+1}$ is increasing ?

A. $(-\infty,-1]\cup[1,\infty)$ B. $[-1,1]$ C. $(-\infty,1]$ D. $[1,\infty)$

56. The radius of a spherical gas balloon is increasing at a rate of $2cm/sec$. At what rate it's surface area increasing when the radius is $3cm$?

A. $52\pi\ cm^2/sec$ B. $48\pi\ cm^2/sec$ C. $32\pi\ cm^2/sec$ D. $16\pi\ cm^2/sec$

57. A rectangular field of $1600m^2$ is to be fenced off. Two opposite sides will fencing costing 2 Birr per meter and the remaining sides will use fencing costing 8 Birr per meter. How much is the least cost of fencing it in Birr? A . 320 B. 420 C. 560 D. 640

58. $\int \left(5x^4 - \frac{2}{\sqrt{x}}\right) dx$ is equal to: A. $\frac{1}{5}x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$ C. $x^5 - 4\sqrt{x} + c, c \in \mathbb{R}$

B. $x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$ D. $\frac{4}{5}x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$

59. $\int_1^3 \left(3x + \frac{1}{x^2}\right) dx$ is equal to: A. $\frac{35}{3}$ B. $\frac{38}{3}$ C. $\frac{34}{3}$ D. $\frac{37}{3}$

60. *What is the area of the region bounded by the graph of* $f(x) = -x^2 + 4x - 3$ *and the x-axis from* $x = 0$ *and* $x = 3$*in square units?*

A. $\frac{8}{3}$ *B.* $\frac{7}{3}$ *C.* $\frac{10}{3}$ *D.* $\frac{11}{3}$

61. The solution region of the system of inequalities $\begin{cases} 2x - y \leq 3 \\ x > 0 \\ y < 0 \end{cases}$ *is found in quadrant* _____

A. One B. Two C. Three D. four

62. The solution set of the system of inequalities $\begin{cases} 2x - y \geq 4 \\ x + y \geq 2 \end{cases}$ is

A. $\{(x, y): x \geq 2 \text{ and } -x + 2 \leq y \leq 2x - 4\}$ C. $\{(x, y): x \geq 2 \text{ and } y \geq 0\}$

B. $\{(x, y): x \geq 0 \text{ and } 2 - x \leq y \leq 2x - 4\}$ D. $\{(x, y): x \geq 0 \text{ and } 2x - 4 \leq y \leq 2 - x\}$

63. The minimum value of the objective function $z = 3x + 5y$

Subject to $\begin{cases} x + 3y \geq 3 \\ x + y \geq 2 \\ x \geq 0, y \geq 0 \end{cases}$ is:

A. 5 B. 6 C. 7 D. 8

64. The corner points of the feasible region determined by the following systems of linear inequalities $y \geq x, x + 3y \leq 60, x + y \geq 10, x \geq 0, y \geq 0$ *are* $(0,20), (15,15)(0,10)$ *and* $(5,5)$. Let $z = 4ax + 2by$, where $a, b > 0$. Condition on a and b so that the maximum of z occurs at both $(15,15)$ *and* $(0,20)$ is: A. $b = 6a$ *B.* $a = 6b$ *C.* $b = 2a$ *D.* $a = 2b$

65. A manufacturer produces two types of nails, $N_1\ and\ N_2$. It takes $1\ hour$ of work on machine A and $3\ hours$ on machine B to produce a package of N_1. It takes $3\ hours$ of work on machine A and $1\ hour$ on machine B to produce a package of N_2. He earns a profit of Birr 17.50 per package on N_1 and Birr 7.00 per package on N_2. How many packages of $N_1\ and\ N_2$, respectively should be $produced$ each day so as to maximize his profit, if he operates his machines for at most $12\ hours\ a\ day$? A. $4\ and\ 4$ B. $3\ and\ 3$ C. $3\ and\ 4$ D. $4\ and\ 3$