دانشگاه علم و صنعت ایران

بنام خدا

امتحان میان ترم – مکانیک محیط پیوسته

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استاد: دکتر طاهری بهروز

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1. Verify the following vector identity is valid if $\vec{t}$ is an arbitrary vector and $\vec{n}$ is a unit normal vector:

$$\vec{t} = (\vec{t}.\vec{n})\vec{n} + \vec{n} \times (\vec{t} \times \vec{n})$$

2. Consider the special temperature field:

$$\theta = \frac{e^{-3t}}{x^2} \qquad \text{where} \qquad x^2 = x_1^2 + x_2^2 + x_3^2$$

Determine the material time derivative $\frac{D\theta}{Dt}$ in terms of spatial coordinates and time for a deformation that is defined by:

$$x_1 = X_1 e^t - X_3(e^t - 1),\; x_2 = X_2 e^{-t} + X_3(1 - e^{-t}),\; x_3 = X_3$$

where x_i and X_i denote the spatial and material coordinates in a Cartesian coordinate system. The time is denoted by t.

3. Consider the motion given by the mapping,

$$x_1 = X_1 + \gamma t X_2,\; x_2 = -\gamma t X_1 + X_2,\; x_3 = X_3$$

where γ is a constant. Also consider the proper orthogonal tensor **Q**, given in matrix form as,

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Can Q ever represent the rotation tensor associated with the polar decomposition of this motion, and if so, under what circumstances?

4. The deformation of a body is defined by,

$$x_1 = X_1 - X_2 + X_3,\; x_2 = X_2 - X_3 + X_1, x_3 = X_3 - X_1 + X_2$$

Determine the ratio between material volumes in the reference and the deformed configuration. Further, consider two line elements N_1, N_2 in the reference configuration with directions,

$$N_1 = \frac{1}{\sqrt{2}} e_1 + \frac{1}{\sqrt{2}} e_2, \qquad N_2 = e_2$$

Where e_1, e_2 denote unite base vectors in the 1-2 directions. Determine the change in angle between the two line elements after deformation.