

ENG KATTA VA ENG KICHIK QIYMAT TOPISHGA OID GEOMETRIK MASALALAR

- 1) Perimetri 10 ga teng bo'lgan to'g'ri to'rtburchak yuzining eng katta qiymatini toping.

$$2(a+b)=10$$

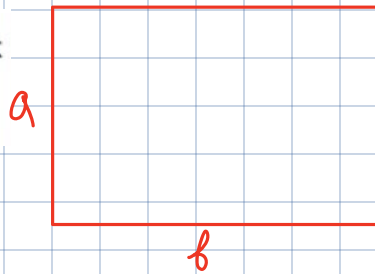
$$a+b=5 \rightarrow b=5-a.$$

$$S=a \cdot b = a \cdot (5-a) = \underline{5a-a^2}.$$

$$S' = 5-2a = 0$$

$$a = \frac{5}{2}.$$

$$S_{\max} = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}.$$



- 2) To'g'ri to'rtburchak shaklidagi yer maydonining uchta tomoni panjara bilan o'ralgan. Agar panjaraning uzunligi 60 ga teng bo'lsa, yer maydoni yuzining eng katta qiymatini toping.

$$2a+b=60.$$

$$S=a \cdot b = a \cdot (60-2a) = \underline{60a-2a^2}.$$

$$S' = 60-4a = 0$$

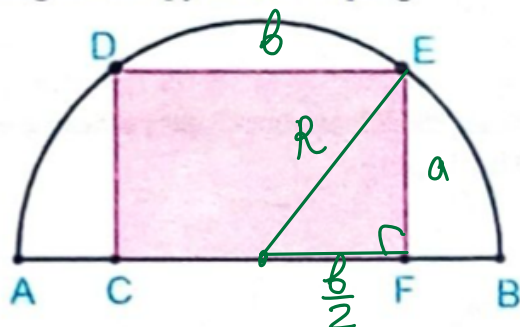
$$a=15.$$

$$S_{\max} = 15 \cdot 30 = 450.$$



3)

Radiusi 6 ga teng bo'lgan yarim aylanaga CDEF to'g'ri to'rtburchak ichki chizilgan. S_{CDEF} ning eng katta qiymatini toping.



$$R=6$$

$$36 = a^2 + \frac{b^2}{4}$$

$$36 - a^2 = \frac{b^2}{4}$$

$$\sqrt{36 - a^2} = \frac{b}{2}$$

$$S = a \cdot b = a \cdot 2\sqrt{36 - a^2} = 2a(36 - a^2)^{\frac{1}{2}}$$

$$S' = 2 \cdot \sqrt{36 - a^2} + 2a \cdot \frac{1}{2} (36 - a^2)^{-\frac{1}{2}} \cdot (-2a) = 0,$$

$$2\sqrt{36 - a^2} - \frac{2a^2}{\sqrt{36 - a^2}} = 0$$

$$2\sqrt{36 - a^2} = \frac{2a^2}{\sqrt{36 - a^2}}$$

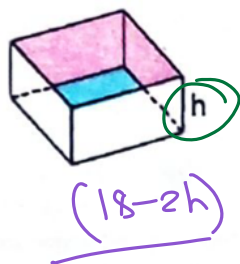
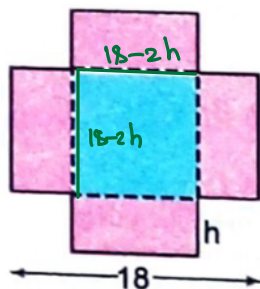
$$36 - a^2 = a^2$$

$$a^2 = 18$$

$$a = 3\sqrt{2}$$

$$S_{\max} = 2 \cdot 3\sqrt{2} \cdot \sqrt{36 - 18} = \underline{\underline{36}}$$

4) Tomoni 18 ga teng bo'lgan kvadratning chetlaridan tomoni h ga teng bo'lgan kvadratchalar qirqib olinib, qolgan qismidan quticha yasaldi. Qutichaning hajmi eng katta bo'lishi uchun qirqib olingan kvadratchalar tomoni qanday bo'lishi kerak?



$$V = abc$$

$$V = (18 - 2h)(18 - 2h) \cdot h$$

$$V = (18 - 2h)(18h - 2h^2)$$

$$V' = -2(18h - 2h^2) + (18 - 2h)(18 - 4h) = 0$$

$$(18 - 2h)(-2h + 18 - 4h) = 0$$

$$18 = 6h$$

$$\underline{\underline{h = 3}}$$

- 5) Radiusi 10 ga teng bo'lgan sferaga ichki chizilgan barcha muntazam uchburchakli prizmalar orasidan hajmi eng katta bo'ladigani tanlab olindi. Shu prizmaning hajmini toping.

$$\frac{h^2}{4} + \frac{a^2}{3} = 100$$

$$\frac{a^2}{3} = 100 - \frac{h^2}{4} \quad | \cdot 3$$

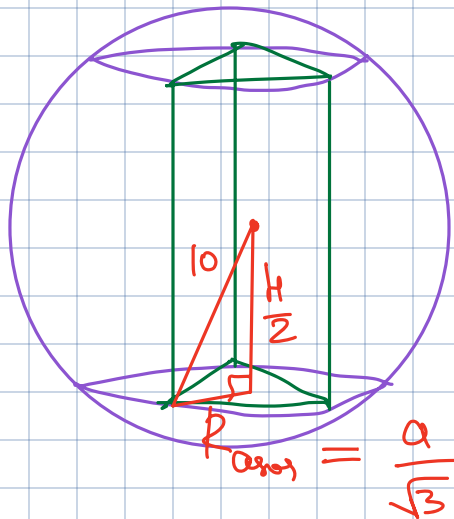
$$a^2 = 300 - \frac{3h^2}{4}$$

$$V = S \cdot h = \frac{\sqrt{3}}{4} a^2 h = \frac{\sqrt{3}}{4} \left(300h - \frac{3h^3}{4} \right)$$

$$V' = \frac{\sqrt{3}}{4} \left(300 - \frac{9h^2}{4} \right) = 0$$

$$300 = \frac{9h^2}{4} \quad h = \frac{20}{\sqrt{3}}$$

$$V_{\max} = \frac{\sqrt{3}}{4} \left(2000\sqrt{3} - \frac{3}{4} \cdot \frac{3000}{\sqrt{3}} \right) = \frac{\sqrt{3}}{4} \cdot \frac{4000}{\sqrt{3}} = 1000$$



- 6) To'la sirti yuzi 96π ga teng bo'lgan silindr hajmining eng katta qiymatini toping.

$$S_T = 2\pi R^2 + 2\pi R h = 96\pi \quad | : 2\pi$$

$$R^2 + R h = 48$$

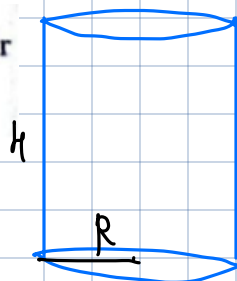
$$h = \frac{48 - R^2}{R}$$

$$V = \pi R^2 h = \pi R^2 \cdot \frac{48 - R^2}{R} = \pi (48R - R^3)$$

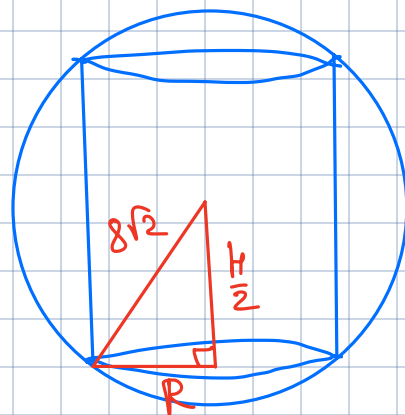
$$V' = \pi (48 - 3R^2) = 0 \Rightarrow 3R^2 = 48$$

$$\underline{R = 4}$$

$$V_{\max} = \pi \cdot 4 \cdot (48 - 16) = 128\pi$$



12) Radiusi $8\sqrt{2}$ ga teng bo'lgan sharga ichki chizilgan va yon sirtining yuzi eng katta bo'lgan silindr asosining radiusini toping.



$$\frac{h^2}{4} + R^2 = 128.$$

$$\frac{h^2}{4} = 128 - R^2.$$

$$h = 2\sqrt{128 - R^2}$$

$$S_{\text{yon}} = 2\pi R h = 4\pi R \sqrt{128 - R^2}$$

$$S' = 4\pi \sqrt{128 - R^2} + 4\pi R \cdot \frac{1}{2} (128 - R^2)^{-\frac{1}{2}} \cdot (-2R) = 0 \quad | : 4\pi.$$

$$\sqrt{128 - R^2} - \frac{R^2}{\sqrt{128 - R^2}} = 0$$

$$128 - R^2 = R^2$$

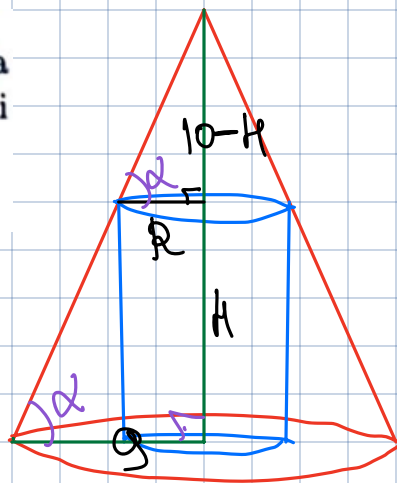
$$R = 8$$

8) Konus asosining radiusi 9 ga, balandligi 10 ga teng. Konusga ichki chizilgan eng katta hajmli silindr hajmini toping.

$$R_k = 9 \quad h_k = 10$$

$$\tan \alpha = \frac{10 - h}{R} = \frac{10}{9}.$$

$$R = \frac{9}{10} (10 - h)$$



$$V_s = \pi R^2 h = \pi \cdot \frac{81}{100} (10 - h)^2 \cdot h.$$

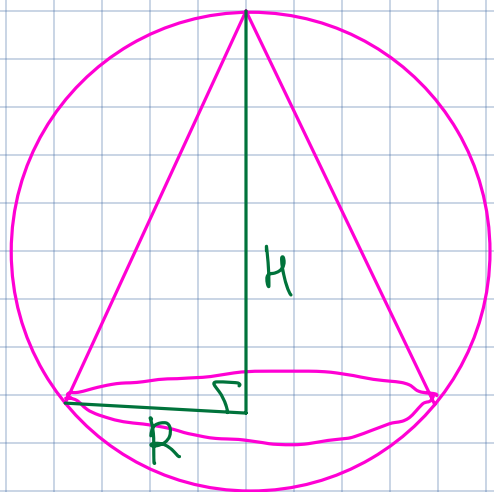
$$V' = \frac{81\pi}{100} \left(2(10 - h) \cdot (-1) \cdot h + (10 - h)^2 \cdot 1 \right) = 0$$

$$\frac{81\pi}{100} \cdot (10 - h) \left(-2h + 10 - h \right) = 0$$

$$V_{\text{max}} = \frac{81\pi}{100} \cdot \frac{400}{9} \cdot \frac{10}{3} = 120\pi.$$

$$10 = 3h \\ h = \frac{10}{3}$$

- 9) Radiusi 12 ga teng bo'lgan sharga ichki chizilgan eng katta hajmli konus balandligini toping.



$$R_{sh} = 12$$

$$R_{sh} = \frac{R^2 + h^2}{2h} = 12$$

$$R^2 = 24h - h^2$$

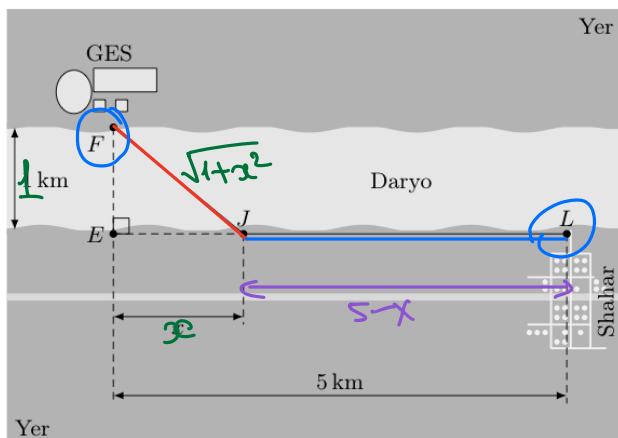
$$V = \frac{\pi}{3} R^2 h = \frac{\pi}{3} (24h - h^2)$$

$$V' = \frac{\pi}{3} (48h - 2h^2) = 0$$

$$48h = 2h^2$$

$$h = 16$$

- 10) Shaharni elektr energiyasi bilan ta'minlash maqsadida ikki xil kabeldan foydalanilgan. Daryo tubidan o'tadigan FJ uzunlikdagi kabelning har bir kilometri uchun 7500\$ dan va qirg'oq bo'ylab (yer ostidan) JL masofaga tortilgan kabelning har bir kilometri uchun 6000\$ dan pul to'langan. Bunda daryoning kengligi $FE=1$ km, $EL=5$ km va $EL \perp FE$ bo'lib, eng kam pul (\$) sarflab F nuqta(stansiya)dan L nuqta(shahar)ga kabel tortib borilgan.



$$f(x) = 7500\sqrt{1+x^2} + 6000(5-x)$$

$$f'(x) = 7500 \cdot \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x - 6000 = 0$$

$$\frac{7500x}{\sqrt{1+x^2}} = 6000 \quad \uparrow^2$$

$$25x^2 = 16 + 16x^2$$

$$9x^2 = 16$$

[1,5 ball]

$$x = \frac{4}{3}$$

- a) Qirg'oq bo'ylab JL masofaga tortilgan kabel uchun qancha mablag' (\$) sarflangan?

Javob: a) _____

[1,7 ball]

- b) Daryo tubidan FJ masofaga tortilgan kabel uchun qancha mablag' (\$) sarflangan?

Javob: b) _____

$$A) (5-x) \cdot 6000 = (5 - \frac{4}{3}) \cdot 6000 = \underline{2000}$$

$$B) 7500 \sqrt{1+x^2} = 7500 \cdot \sqrt{1 + \frac{16}{9}} = 7500 \cdot \frac{5}{3} = \underline{12500}$$



