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TRIGONOMETRIYADAN MASALALAR YECHISH

Oliy o'quv yurtlariga kiruvchilar uchun metodik qo'llanma

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Ushbu qo'llanmada Oliy o'quv yurtlariga kirish imtihonlarida berilgan trigonometrik ifodalarning qiymatini hisoblash, soddalashtirish, tenglamalar va tengsizliklarga doir qiyin va murakkab masalalarning yechilishi ko'rsatma va izohlar bilan bayon qilingan.

Qo'llanma oliy o'qiv yurtlariga kirishni maqsad qilgan barcha yoshlarga, umumiy o'rta ta'lim maktablari, litsey va kollej o'quvchilariga mo'ljallangan, shuningdek, matematika o'qituvchilarining ish faoliyatida eng yaqin yordamchi hamdir.

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SO‘Z BOSHI

Ushbu qo‘llanmada Oliy o‘quv yurtlariga kirish imtihonlarida trigonometrik ifodalarning qiymatini topish, soddalashtirish, tenglamalar va tengsizliklarni yechishga doir turli masalalar bayon qilingan. Ular matematikaning aynan shu yo‘nalishi bo‘yicha abituriyentlarning bilimini sinovdan o‘tkazishga mo‘ljallangan.

Ko‘p yillik pedagogik tajribadan shu narsa ma’lumki, o‘quvchilar trigonometriyaga oid bilim va ko‘nikmalarni egallashda ancha qiynaladi. Ana shuni nazarda tutib ushbu metodik qo‘llanmada o‘quvchilarning trigonometrik tenglamalar, ularning turlari va yechish usullari, teskari trigonometrik funksiyalar qatnashgan tenglamalar va ularni yechish hamda trigonometrik tengsizliklar va ularni yechishning oson va qulay usullarini bayon qilishda muhtaram o‘quvchi tez o‘zlashtirib oladigan qirralarga e’tibor qaratishga harakat qilindi.

Trigonometrik ifodalarni qiymatini topish, soddalashtirish o‘quvchidan qisqa ko‘paytirish formulalarini, trigonometrik formulalarni va trigonometrik funksiyalarning qiymatlar jadvalini puxta bilishni talab qilsa, trigonometrik tenglama va tengsizliklarni yechish esa trigonometrik funksiyalarning xossalarini (aniqlanish va qiymatlar sohasini, o‘sish va kamayish oraliqlarini, davriyligini, juft-toqligini, eng katta va eng kichik qiymatini, nollarini va h.k.), tenglamani (tengsizlikni), eng sodda trigonometrik tenglama ko‘rinishiga keltirib, yechimlarini (yechimlar to‘plamini) topish formulalarini bilishni, tenglamani yechish jarayonida ildizlarning yo‘qolib ketish hollarini yoki begona ildizlarning paydo bo‘lib qolish hollarini tahlil qila bilishni talab qiladi.

Shuning uchun qo‘llanmada asosiy trigonometrik formulalar va ayrim mavzular bo‘yicha qisqacha ma’lumotlar berish bilan birga to‘plamdagi barcha masalalarning yechilishlarini aniq va ratsional usullarda, o‘quvchilar tilidan bayon qilishga harakat qilindi.

To‘plamni yozishdan maqsad Oliy o‘quv yurtlariga kirish uchun test sinovlarida tushgan barcha masalalarni yechib ko‘rsatish emas, balki kirish imtihonlarida berilgan abituriyent uchun qiyin va murakkab tuyulgan, nostandart usullarda yechiladigan, mantiqiy fikr-mulohazalar yuritishni talab qiladigan va ko‘proq uchraydigan ayrim masalalarning yechilishidan namunalar berish bilan muhtaram o‘quvchiga shu tipdagi masalani yechishda to‘g‘ri yo‘lni tanlay bilishda ko‘maklashishdir.

Agar ushbu qo‘llanma kimgadir yecholmayotgan masalasini yechishda yoki kimningdir talaba bo‘lishdek orzu-umidini ro‘yobga chiqarishda yordam bera olsa, bu bizning katta yutug‘imizdir.

Mualliflar

TRIGONOMETRIK IFODALARNING QIYMATINI HISOBLASHDA, SODDALASHTIRISHDA HAMDA TRIGONOMETRIK TENGLAMALARNI VA TENGSIZLIKLARNI YECHISHDA QO'LLANILADIGAN ASOSIY FORMULALAR

1. Qisqa ko'paytirish formulalari:

$$(a+b)^2 = a^2 + 2ab + b^2, \qquad (1)$$

$$(a-b)^2 = a^2 - 2ab + b^2, \qquad (2)$$

$$a^2 - b^2 = (a-b)(a+b), \qquad (3)$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3, \qquad (4)$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3, \qquad (5)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2), \qquad (6)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \qquad (7)$$

2. Trigonometrik funksiyalarning choraklardagi ishoralari:

Choraklar	$\sin x$	$\cos x$	$\operatorname{tg} x$	$\operatorname{ctg} x$
I	+	+	+	+
II	+	–	–	–
III	–	–	+	+
VI	–	+	–	–

3. Trigonometrik funksiyalarning ba'zi qiymatlari:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–
$\operatorname{ctg} x$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0

4. Trigonometrik funksiyalarning juft-toqligi.

$y = \cos x$ – juft funksiya, ya'ni $\cos(-x) = \cos x$.

$y = \sin x,\ \ y = tgx,\ \ y = ctgx$ – toq funksiyalar, ya'ni

$\sin(-x) = -\sin x$, $tg(-x) = -tgx$, $\left(x \neq \frac{\pi}{2} + \pi n,\ \ n \in Z\right)$

$ctg(-x) = -ctgx$, $(x \neq \pi n,\ \ n \in Z)$

5. Trigonometrik funksiyalarning davriyligi.

Barcha trigonometrik funksiyalar davriydir.

$y = \sin x$ va $y = \cos x$ funksiyalarning eng kichik musbat davri 2π ga, $y = tgx$ va $y = ctgx$ funksiyalarning eng kichik musbat davri π ga teng, ya'ni

$$\sin(x + 2\pi) = \sin(x - 2\pi) = \sin x,$$
$$\cos(x + 2\pi) = \cos(x - 2\pi) = \cos x,$$
$$tg(x + \pi) = tg(x - \pi) = tgx, \quad \left(x \neq \frac{\pi}{2} + \pi n,\ \ n \in Z\right)$$
$$ctg(x + \pi) = ctg(x - \pi) = ctgx, \quad (x \neq \pi n,\ \ n \in Z)$$

6. Bir xil argumentli trigonometrik funksiyalar orasidagi bog'lanishlar formulalari:

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad (9)$$

$$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}, \left(\alpha \neq \frac{\pi}{2} + \pi n,\ \ n \in Z\right) \qquad (10)$$

$$1 + ctg^2\alpha = \frac{1}{\sin^2\alpha}, (\alpha \neq \pi n,\ \ n \in Z) \qquad (11)$$

$$tg\alpha \cdot ctg\alpha = 1, \qquad (12)$$

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha}, \qquad (13)$$

$$ctg\alpha = \frac{\cos\alpha}{\sin\alpha}. \qquad (14)$$

7. Ikkilangan burchakning trigonometrik funksiyalari formulalari

$$\sin 2\alpha = 2\sin\alpha\cos\alpha, \qquad (15)$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha, \qquad (16)$$

$$tg\,2\alpha = \frac{2tg\,\alpha}{1 - tg^2\alpha}, \quad \left(\alpha \neq \frac{\pi}{4} + \frac{\pi n}{2},\ \alpha \neq \frac{\pi}{2} + \pi k,\ n \in Z\right) \qquad (17)$$

$$ctg\,2\alpha = \frac{ctg^2\alpha - 1}{2ctg\,\alpha}, \quad \left(\alpha \neq \frac{\pi n}{2},\ n \in Z\right) \qquad (18)$$

$$1 + \cos 2\alpha = 2\cos^2\alpha, \qquad (19)$$

$$1 - \cos 2\alpha = 2\sin^2\alpha, \qquad (20)$$

$$1 \pm \sin 2a = (\cos\alpha \pm \sin\alpha)^2 \qquad (21)$$

8. Argumentlar yig'indisi formulalari:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha, \qquad (22)$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\beta\sin\alpha, \qquad (23)$$

$$tg(\alpha \pm \beta) = \frac{tg\alpha \pm tg\beta}{1 \mp tg\alpha \cdot tg\beta}, \qquad (24)$$

$$ctg(\alpha \pm \beta) = \frac{ctg\alpha \cdot ctg\beta \mp 1}{ctg\beta \pm ctg\alpha}, \qquad (25)$$

9. Keltirish formulalari:

x	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$\sin x$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos x$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
tgx	$ctg\alpha$	$-ctg\alpha$	$-tg\alpha$	$tg\alpha$	$ctg\alpha$	$-ctg\alpha$	$-tg\alpha$
$ctgx$	$tg\alpha$	$-tg\alpha$	$-ctg\alpha$	$ctg\alpha$	$tg\alpha$	$-tg\alpha$	$-ctg\alpha$

10. Trigonometrik funksiyalar yig'indisini ko'paytmaga almashtirish formulalari:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad (26)$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}, \qquad (27)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad (28)$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}, \qquad (29)$$

$$tg\alpha \pm tg\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cdot\cos\beta}, \qquad (30)$$

$$ctg\alpha \pm ctg\beta = \frac{\sin(\beta\pm\alpha)}{\sin\alpha\cdot\sin\beta}, \qquad (31)$$

$$\cos\alpha \pm \sin\alpha = \sqrt{2}\cos\left(\frac{\pi}{4}\mp\alpha\right) \qquad (32)$$

11. Trigonometrik funksiyalarning ko'paytmasini yig'indiga almashtirish formulalari:

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)], \qquad (33)$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)], \qquad (34)$$

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \qquad (35)$$

12. Burchakning radian va gradus o'lchovlari orasidagi bog'lanish:

$$\alpha^\circ = \frac{180^\circ}{\pi}\cdot\alpha_{rad}$$ – gradusga o'tish. (36)

$$\alpha_{rad} = \frac{\pi}{180^\circ}\cdot\alpha^\circ$$ – radianga o'tish. (37)

Ko'pincha $\sin\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot ...\cdot\cos 2^n\alpha$ yoki $\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot ...\cdot\cos 2^n\alpha$ ko'paytma ko'rinishidagi trigonometrik ifodalarning qiymatlarini topishda yoki soddalashtirishda berilgan ifodani $\cos\alpha$ yoki $\sin\alpha$ ga ham ko'paytirib, ham bo'linadi va $2\sin\alpha\cos\alpha=\sin 2\alpha$ formuladan foydalanishga keltiriladi.

Misollar. 1. $\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha$ ni soddalashtiring.

Yechilishi. Berilgan ifodani $2\sin\alpha$ ga ko'paytirib bo'lamiz.

$$\frac{2\sin\alpha\cdot\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{2\sin\alpha},$$

Bu yerda $2\sin\alpha\cos\alpha=\sin 2\alpha$ bo'lganidan, o'rniga qo'yib

$$\frac{\sin 2\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{2\sin\alpha}$$ ifodani hosil qilamiz.

Endi kasrning surat va maxrajini ketma-ket to'rt marta 2 ga ko'paytirib bo'lish bilan natijaga erishamiz.

$$\frac{2\sin 2\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{4\sin\alpha}=\frac{\sin 4\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{4\sin\alpha},$$

$$\frac{2\sin 4\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{8\sin\alpha}=\frac{\sin 8\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{8\sin\alpha},$$

$$\frac{2\sin 8\alpha\cdot\cos 8\alpha\cdot\cos 16\alpha}{16\sin\alpha}=\frac{\sin 16\alpha\cdot\cos 16\alpha}{16\sin\alpha},$$

$$\frac{2\sin 16\alpha\cdot\cos 16\alpha}{32\sin\alpha}=\frac{\sin 32\alpha}{32\sin\alpha}.$$

2. $\sin 47^\circ+\sin 61^\circ-\sin 11^\circ-\sin 25^\circ$ yig'indini toping.

Yechilishi. $(\sin 47^\circ+\sin 61^\circ)-(\sin 11^\circ+\sin 25^\circ)$ deb guruhlab olib, (26) formuladan foydalanib, ko'paytma ko'rinishiga keltiramiz.

$$2\sin 54^\circ\cdot\cos 7^\circ-2\sin 18^\circ\cdot\cos 7^\circ=2\cos 7^\circ(\sin 54^\circ-\sin 18^\circ).$$

U holda (27) formulaga asosan

$2\cos 7^\circ\cdot 2\sin 18^\circ\cdot\cos 36^\circ$ ifoda hosil bo'lib, buni $\cos 18^\circ$ ga ko'paytirib, ham bo'lib

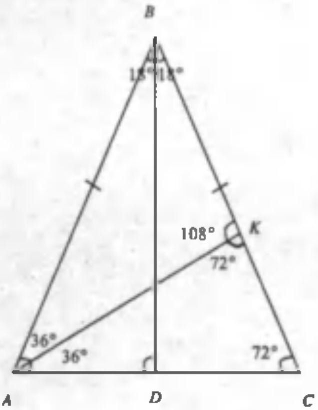
$$\frac{2\cos 7^\circ\cdot 2\sin 18^\circ\cdot\cos 18^\circ\cdot\cos 36^\circ}{\cos 18^\circ}=\frac{2\cos 7^\circ\cdot\sin 36^\circ\cdot\cos 36^\circ}{\cos 18^\circ}=$$

$$=\frac{\cos 7^\circ\cdot\sin 72^\circ}{\cos 18^\circ}=\cos 7^\circ\cdot\frac{\sin(90^\circ-18^\circ)}{\cos 18^\circ}=\cos 7^\circ\cdot\frac{\cos 18^\circ}{\cos 18^\circ}=\cos 7^\circ$$ ni hosil qilamiz.

TRIGONOMETRIK IFODALARNING QIYMATINI HISOBLASH BA SODDALASHTIRISHGA DOIR MASALALARNI YECHISH

1. $\sin 18^\circ$ ni hisoblang.

1-usul.



Uchidagi burchagi 36° va yon tomoni 1 ga teng bo'lgan teng yonli uchburchakni qaraymiz.

AC=*x* desak, to'g'ri burchakli ΔABD dan $\sin 18^\circ = \frac{x}{2}$ bo'ladi. Endi *ABC* uchburchakning asosi *AC*=*x* ni topsak, masala hal bo'ladi.

A dan *BC* ga *AK* bissektrisani o'tkazamiz. Natijada $\Delta ABC \backsim \Delta AKC$ lar hosil bo'ladi (2 ta burchagi bo'yicha). *AC*=*AK*=*BK*=*x* va *CK*=1-*x* bo'ladi.

$\Delta ABC \backsim \Delta AKC$ dan

$\frac{AC}{CK} = \frac{AB}{AC} \Rightarrow \frac{x}{1-x} = \frac{1}{x} \Rightarrow x^2 + x - 1 = 0$ kvadrat tenglama hosil bo'ladi. Uni yechib $x_1 = \frac{\sqrt{5}-1}{2}$ va $x_2 = \frac{-\sqrt{5}-1}{2}$ ni hosil qilamiz.

$\sin 18^\circ = \frac{x}{2} = \frac{\sqrt{5}-1}{2}$, bunda $x_2 < -1$ bo'lganidan masala shartini qanoatlantirmaydi.

2-usul.

$\cos 36° = \sin 54°$

$\cos(2 \cdot 18°) = \sin(3 \cdot 18°)$

$1 - 2\sin^2 18° = 3\sin 18° - 4\sin^3 18°$

$\sin 18° = a$ belgilasak,

$1 - 2a^2 = 3a - 4a^3$

$4a^3 - 2a^2 - 3a + 1 = 0$

$a = 1$ tenglama ildizi, shuning uchun tenglikning chap tomonidagi ifoda $a - 1$ ga qoldiqsiz bo'linadi.

$(a - 1)(4a^2 - 2a - 1) = 0$

1) $a_1 = 1$ 2) $4a^2 + 2a - 1 = 0$

$$a_2 = \frac{-2 + \sqrt{20}}{8} = \frac{-2 + 2\sqrt{5}}{8} = \frac{\sqrt{5} - 1}{4} \qquad a_3 = \frac{-\sqrt{5} - 1}{4}$$

a_1 va a_3 lar chet ildizlar (o'ylab ko'ring).

$$\sin 18° = \frac{\sqrt{5} - 1}{4}$$

2. $\cos 15°$ va $\sin 15°$ ni hisoblang.

1-usul. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ va $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ formulalardan foydalanamiz.

$$\cos 15° = \cos(60° - 45°) = \cos 60° \cdot \cos 45° + \sin 60° \cdot \sin 45° = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\sin 15° = \sin(60° - 45°) = \sin 60° \cdot \cos 45° - \cos 60° \cdot \sin 45° = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Bundan tashqari 15°=45°–30° deb ham $\cos 15°$ va $\sin 15°$ ning qiymatlarini hosil qilish mumkin (mustaqil bajarib ko'ring).

2-usul. $\cos\alpha = \pm\sqrt{\frac{1 + \cos 2\alpha}{2}}$ va $\sin\alpha = \pm\sqrt{\frac{1 - \cos 2\alpha}{2}}$ darajani pasaytirish formulalari yordamida. Bunda $\alpha = 15° > 0$

$$\cos 15° = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\sin 15^\circ = \sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

Shu o'rinda $\frac{\sqrt{6}+\sqrt{2}}{4}$ va $\frac{\sqrt{2+\sqrt{3}}}{2}$ hamda $\frac{\sqrt{6}-\sqrt{2}}{4}$ va $\frac{\sqrt{2-\sqrt{3}}}{2}$ sonli ifodalarning bir-biriga tengligini ko'rsatish mumkin.

3. $\cos 18^\circ$ ni hisoblang.

Yechilishi. $\cos\alpha = \sqrt{1-\sin^2\alpha}$ $(\alpha = 18^\circ > 0)$ formuladan foydalanamiz.

$$\cos 18^\circ = \sqrt{1-\sin^2 18^\circ} = \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2} = \sqrt{1-\frac{6-2\sqrt{5}}{16}} = \sqrt{\frac{10+2\sqrt{5}}{16}} = \frac{1}{4}\sqrt{10+2\sqrt{5}}$$

Demak, $\cos 18^\circ = \frac{1}{4}\sqrt{10+2\sqrt{5}}$ ga teng ekan.

Oliy o'quv yurtlariga kirish imtihonlarida berilgan masalalar

1. (01-2-86). Agar $\sin 2x = \frac{2}{5}$ bo'lsa, $\sin^8 x + \cos^8 x$ ning qiymatini toping.

A) $\frac{16}{25}$ B) $\frac{398}{625}$) $\frac{527}{625}$ D) $\frac{256}{625}$ E) $\frac{8}{25}$

Yechilishi:

$$\sin^8 x + \cos^8 x = (\sin^4 x)^2 + (\cos^4 x)^2 = (\sin^4 x)^2 + (\cos^4 x)^2 + 2\sin^4 x\cos^4 x - 2\sin^4 x\cdot\cos^4 x =$$

$$= (\sin^4 x - \cos^4 x)^2 + 2\sin^4 x\cos^4 x = (\sin^2 x - \cos^2 x)^2\cdot(\sin^2 x + \cos^2 x)^2 + 2\sin^4 x\cdot\cos^4 x =$$

$$= \cos^2 2x + \frac{(2\sin x\cos x)^4}{8} = \cos^2 2x + \frac{(\sin 2x)^4}{8} = (1-\sin^2 2x)^2 + \frac{(\sin 2x)^4}{8} = 1-\frac{4}{25} + \frac{\left(\frac{2}{5}\right)^4}{8} =$$

$$= \frac{21}{25} + \frac{6}{625\cdot 8} = \frac{21}{25} + \frac{2}{625} = \frac{527}{625}$$

Javob: C.

2. (01-3-3). Hisoblang $\sin^4 15^0 + \cos^4 15^0$.

A) $\frac{5}{6}$ B) $\frac{2}{3}$ C) $\frac{7}{8}$ D) $\frac{5}{7}$ E) $\frac{2}{7}$

Yechilishi:

$$\sin^4 15^0+\cos^4 15^0=(\sin^2 15^0)^2+(\cos^2 15^0)^2=(\sin^2 15^0)^2+2\sin^2 15^0\cdot\cos^2 15^0+(\cos^2 15^0)^2-$$

$$-2\sin^2 15^0\cos^2 15^0=(\sin^2 15^0+\cos^2 15^0)^2-\frac{1}{2}(2\sin 15^0\cdot\cos 15^0)^2=1-\frac{1}{2}(\sin 30^0)^2=$$

$$=1-\frac{1}{2}\left(\frac{1}{2}\right)^2=1-\frac{1}{8}=\frac{7}{8}.$$

Javob: C.

(Hisoblash davomida $\sin^2 x+\cos^2 x=1$, $\sin 2x=2\sin x\cdot\cos x$ formulalardan foydalandik va $(a+в)^2$ ga tushirish uchun $2\sin^2 15^0\cos^2 15^0$ ni *qo'shib, ayirdik*).

3. (01-5-15). $tg10^0\cdot tg50^0\cdot tg70^0$ ni hisoblang.

A) $\frac{1}{\sqrt{3}}$ B) $\sqrt{3}$ C) 0 D) 1 E) $\frac{1}{\sqrt{2}}$

Yechilishi: 1- usul: $tgx=\frac{\sin x}{\cos x}$ dan foydalanib, berilgan ifodani

$$tg10^0\cdot tg50^0\cdot tg70^0=\frac{\sin 10^0\cdot\sin 50^0\cdot\sin 70^0}{\cos 10^0\cdot\cos 50^0\cdot\cos 70^0}$$

ko'rinishda yozib olamiz.

So'ngra $(\sin 50^0\cdot\sin 70^0)$ uchun $\sin x\cdot\sin y=\frac{1}{2}(\cos(x-y)-\cos(x+y))$ formulani qo'llab va $(\cos 50^0\cdot\cos 70^0)$ uchun $\cos x\cdot\cos y=\frac{1}{2}(\cos(x-y)+\cos(x+y))$ formulani qo'llab quyidagi ko'rinishda yozib olamiz.

$$\frac{\sin 10^0\cdot\sin 50^0\cdot\sin 70^0}{\cos 10^0\cdot\cos 50^0\cdot\cos 70^0}=\frac{\sin 10^0\cdot\frac{1}{2}(\cos 20^0-\cos 120^0)}{\cos 10^0\cdot\frac{1}{2}(\cos 20^0+\cos 120^0)}=$$

$$=\frac{\sin 10^0(\cos 20^0-\cos(90^0+30^0))}{\cos 10^0(\cos 20^0+\cos(90^0+30^0))}=\frac{\sin 10^0(\cos 20^0+\cos 30^0)}{\cos 10^0(\cos 20^0-\sin 30^0)}=$$

$$=\frac{\sin 10^0\left(\cos 20^0+\frac{1}{2}\right)}{\cos 10^0\left(\cos 20^0-\frac{1}{2}\right)}=\frac{\sin 10^0\cdot\cos 20^0+\frac{1}{2}\sin 10^0}{\cos 20^0\cdot\cos 10^0-\frac{1}{2}\cos 10^0}$$

Endi $\sin x\cdot\cos y=\frac{1}{2}(\sin(x-y)+\sin(x+y))$ formuladan foydalanib,

$\sin 10^0\cdot\cos 20^0=\frac{1}{2}\left(-\sin 10^0+\sin 30^0\right)=-\frac{1}{2}\sin 10^0+\frac{1}{4}$ va

$\cos 10^0\cdot\cos 20^0=\frac{1}{2}\left(\cos 10^0+\cos 30^0\right)=\frac{1}{2}\cos 10^0+\frac{\sqrt{3}}{4}$ larni o'rniga qo'yib, natijaga erishamiz.

$$\frac{\sin 10^0\cdot\cos 20^0+\frac{1}{2}\sin 10^0}{\cos 20^0\cdot\cos 10^0-\frac{1}{2}\cos 10^0}=\frac{-\frac{1}{2}\sin 10^0+\frac{1}{4}+\frac{1}{2}\sin 10^0}{\frac{1}{2}\cos 10^0+\frac{\sqrt{3}}{4}-\frac{1}{2}\cos 10^0}=\frac{\frac{1}{4}}{\frac{\sqrt{3}}{4}}=\frac{1}{\sqrt{3}}.$$

2-usul: $tg3\alpha=tg\alpha\cdot tg\left(60^0-\alpha\right)\cdot tg\left(60^0+\alpha\right)$ formuladan foydalanib yechish mumkin.

$tg\left(3\cdot 10^0\right)=tg10^0\cdot tg\left(60^0-10^0\right)\cdot tg\left(60^0+10^0\right)$

$tg30^0=tg10^0\cdot tg50^0\cdot tg70^0$

Demak, yuqoridagi ifoda $tg30^0$ ga, ya'ni $\frac{1}{\sqrt{3}}$ ga teng ekan.

Javob: A.

4. (01-6-27). Hisoblang $\cos 15^0+\sqrt{3}\sin 15^0$

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{2}}{4}$

Yechilishi: Bunday ko'rinishdagi misollarni yechishda berilgan ifodani ko'p hollarda $\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$ yoki $\cos(x\pm y)=$ $=\cos x\cos y\mp\sin x\sin y$ formulalardan foydalanishga keltirish qulaydir. Shuning uchun berilgan ifodani

$$\cos 15^0+\sqrt{3}\sin 15^0=2\left(\frac{1}{2}\cos 15^0+\frac{\sqrt{3}}{2}\sin 15^0\right)$$

ko'rinishda yozib olamiz. So'ngra $\frac{1}{2}=\sin 30^0$ va $\frac{\sqrt{3}}{2}=\cos 30^0$ ekanidan bu qiymatlarni o'rniga qo'yamiz.

Bunda $2(\sin 30^0 \cos 15^0+\cos 30^0 \sin 15^0)$ hosil bo'ladi. Qavs ichidagi ifoda yuqorida keltirilgan birinchi formulaga asosan $\sin(30^0+15^0)$ga teng bo'ladi. Shunday qilib,

$$\cos 15^0+\sqrt{3}\sin 15^0=2\left(\frac{1}{2}\cos 15^0+\frac{\sqrt{3}}{2}\sin 15^0\right)=2(\sin 30^0\cos 15^0+\cos 30^0\sin 15^0)=$$

$$=2\sin(30^0+15^0)=2\sin 45^0=2\cdot\frac{\sqrt{2}}{2}=\sqrt{2}$$

Javob: B

5. (01-10-35). Ifodaning qiymatini toping. $\cos^8 22^0 30'-\sin^8 22^0 30'$

A) $\frac{\sqrt{2}}{4}$ B) $\frac{\sqrt{2}}{8}$ C) $\frac{3\sqrt{2}}{8}$ D) $\frac{5\sqrt{2}}{8}$ E) $\frac{3\sqrt{2}}{4}$

Yechilishi: Ifodaning qiymatini topish uchun $a^{2n}-в^{2n}=(a^n-в^n)(a^n+в^n)$ formulani qo'llab, soddalashtirishga harakat qilamiz.

$$\cos^8 22^0 30'-\sin^8 22^0 30'=(\cos^4 22^0 30'-\sin^4 22^0 30')\cdot(\cos^4 22^0 30'+\sin^4 22^0 30')=$$

$$=(\cos^2 22^0 30'+\sin^2 22^0 30')\cdot(\cos^2 22^0 30'-\sin^2 22^0 30')(\cos^4 22^0 30'+\sin^4 22^0 30')=$$

$$=\cos 45^0\cdot(\cos^4 22^0 30'+2\cos^2 22^0 30'\cdot\sin^2 22^0 30'+\sin^4 22^0 30'-2\cos^2 22^0 30'\cdot\sin^2 22^0 30')=$$

$$=\cos 45^0\cdot\left((\cos^2 22^0 30'+\sin^2 22^0 30')^2-\frac{(\sin 45^0)^2}{2}\right)=\frac{\sqrt{2}}{2}\left(1-\frac{1}{4}\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{8}=\frac{3\sqrt{2}}{8}$$

Javob: C.

(Hisoblash jarayonida $\sin^2 x+\cos^2 x=1$, $\sin 2x=2\sin x\cdot\cos x$ va $\cos 2x=\cos^2 x-\sin^2 x$ formulalarini qo'llash bilan birga $\cos 45^0=\frac{\sqrt{2}}{2}$, $\sin 45^0=\frac{\sqrt{2}}{2}$ ekanidan ham foydalandik).

6. (01-11-18). Ushbu $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}$ ifodaning qiymatini toping.

A) 3,5 B) 2,5 C) 3 D) 4 E) 4,5

Yechilishi:

$$\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=\frac{\cos 10^0-\sqrt{3}\sin 10^0}{\sin 10^0\cdot\cos 10^0}=\frac{2\left(\frac{1}{2}\cos 10^0-\frac{\sqrt{3}}{2}\sin 10^0\right)}{\frac{2\sin 10^0\cdot\cos 10^0}{2}}=$$

$$=\frac{4\left(\sin 30^0\cos 10^0-\cos 30^0\sin 10^0\right)}{\sin 20^0}=\frac{4\sin\left(30^0-10^0\right)}{\sin 20^0}=\frac{4\sin 20^0}{\sin 20^0}=4$$

Javob: D.

(Misolning yechilishini tushunish uchun 4-misolning yechilishiga qarang)

7. (02-5-34). $\frac{\cos^2 68^0-\cos^2 38^0}{\sin 106^0}$ ni hisoblang.

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) -1

Yechilishi: Ifodaning qiymatini hisoblash uchun kasrning suratini $a^2-в^2=(a-в)(a+в)$ formula yordamida ko'paytuvchilarga ajratib, so'ngra $\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$ va $\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ formulalardan foydalanib, tegishli shakl almashtirishlarni bajaramiz.

$$\frac{\cos^2 68^0-\cos^2 38^0}{\sin 106^0}=\frac{\left(\cos 68^0-\cos 38^0\right)\left(\cos 68^0+\cos 38^0\right)}{\sin 106^0}=$$

$$=\frac{-2\sin\frac{68^0+38^0}{2}\cdot\sin\frac{68^0-38^0}{2}\cdot 2\cos\frac{68^0+38^0}{2}\cdot\cos\frac{68^0-38^0}{2}}{\sin 106^0}=$$

$$=\frac{-4\sin 53^0\cdot\sin 15^0\cdot\cos 53^0\cdot\cos 15^0}{\sin 106^0}=\frac{-\left(2\sin 53^0\cdot\cos 53^0\right)\left(2\sin 15^0\cdot\cos 15^0\right)}{2\sin 53^0\cdot\cos 53^0}=$$

$$=-2\sin 15^0\cdot\cos 15^0=-\sin 30^0=-\frac{1}{2}$$

Javob: B.

8. (03-1-33). $1-\sin^6 22{,}5^0+\cos^6 22{,}5^0$ ni hisoblang.

A) $\frac{\sqrt{3}-1}{2}$ B) $\frac{\sqrt{6}+5}{2}$ C) $\frac{10+3\sqrt{2}}{8}$ D) $\frac{16+7\sqrt{2}}{16}$ E) $\frac{10+2\sqrt{3}}{5}$

Yechilishi: Berilgan ifodaning qiymatini hisoblashda $a^3-в^3=(a-в)(a^2+aв+в^2)$, $\sin^2 x+\cos^2 x=1$, $\cos^2 x-\sin^2 x=\cos 2x$ va $2\sin x\cdot\cos x=\sin 2x$ formulalardan foydalanamiz.

$$1-\sin^6 22{,}5^0+\cos^6 22{,}5^0=1+\cos^6 22{,}5^0-\sin^6 22{,}5^0=1+(\cos^2 22{,}5^0)^3-(\sin^2 22{,}5^0)^3=$$

$$=1+(\cos^2 22{,}5^0-\sin^2 22{,}5^0)(\cos^4 22{,}5^0+\cos^2 22{,}5^0\cdot\sin^2 22{,}5^0+\sin^4 22{,}5^0)=$$

$$=1+\cos 45^0((\cos^4 22{,}5^0+2\cos^2 22{,}5^0\cdot\sin^2 22{,}5^0+\sin^4 22{,}5^0)-\cos^2 22{,}5^0\cdot\sin^2 22{,}5^0)=$$

$$=1+\frac{\sqrt{2}}{2}\left((\cos^2 22{,}5^0+\sin^2 22{,}5^0)^2-\frac{4\sin^2 22{,}5^0\cdot\cos^2 22{,}5^0}{4}\right)=1+\frac{\sqrt{2}}{2}\left(1-\frac{(\sin 45^0)^2}{4}\right)=$$

$$=1+\frac{\sqrt{2}}{2}\left(1-\frac{\left(\frac{\sqrt{2}}{2}\right)^2}{4}\right)=1+\frac{\sqrt{2}}{2}\left(1-\frac{1}{8}\right)=1+\frac{\sqrt{2}}{2}\cdot\frac{7}{8}=1+\frac{7\sqrt{2}}{16}=\frac{16+7\sqrt{2}}{16}$$

Javob: D

9. (03-1-48). $tg555^0$ ni hisoblang.

A) $\frac{\sqrt{3}}{6}$ B) $\sqrt{3}-1$ C) $2-\sqrt{3}$ D) $2+\sqrt{3}$ E) $1-\frac{\sqrt{3}}{2}$

Yechilishi:

$$tg555^0=tg\frac{1110^0}{2}=\frac{1-\cos 1110^0}{\sin 1110^0}=\frac{1-\cos(1080^0+30^0)}{\sin(1080^0+30^0)}=\frac{1-\cos 30^0}{\sin 30^0}=\frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=2-\sqrt{3}$$

Javob: C

(Bu yerda $tg\frac{x}{2}=\frac{1-\cos x}{\sin x}$ formuladan foydalanib, $tg\frac{1110^0}{2}$ ni $\frac{1-\cos 1110^0}{\sin 1110^0}$ orqali ifodaladik, so'ngra keltirish formulalaridan foydalandik, ya'ni $\cos 1110^0=\cos(1080^0+30^0)=\cos 30^0$; $\sin 1110^0=\sin(1080^0+30^0)=\sin 30^0$.

10. (03-2-26). Agar $ctg\alpha = \sqrt{2} - 1$ bo'lsa, $\cos 2\alpha$ ning qiymatini toping.

A) $\sqrt{2}$ B) $\frac{\sqrt{2}+1}{2}$ C) $-\frac{1}{\sqrt{2}}$ D) $-\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

Yechilishi: $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$, $\cos\alpha = \sqrt{\frac{1+\cos 2\alpha}{2}}$ va $\sin\alpha = \sqrt{\frac{1-\cos 2\alpha}{2}}$ formulalardan foydalanib, $\cos 2\alpha$ ning qiymatini topamiz.

$$ctg\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\sqrt{\frac{1+\cos 2\alpha}{2}}}{\sqrt{\frac{1-\cos 2\alpha}{2}}} = \sqrt{\frac{1+\cos 2\alpha}{1-\cos 2\alpha}}$$

Shartga ko'ra, $\sqrt{\frac{1+\cos 2\alpha}{1-\cos 2\alpha}} = \sqrt{2} - 1$. Bu tenglikning ikkala tomonini kvadratga ko'tarib, so'ngra ixchamlab

$\frac{1+\cos 2\alpha}{1-\cos 2\alpha} = 3 - 2\sqrt{2}$, $1+\cos 2\alpha = (3-2\sqrt{2})(1-\cos 2\alpha)$, $1+\cos 2\alpha =$
$= 3 - 3\cos 2\alpha - 2\sqrt{2} + 2\sqrt{2}\cos 2\alpha$. $4\cos 2\alpha - 2\sqrt{2}\cos 2\alpha = 2 - 2\sqrt{2}$;

$(4-2\sqrt{2})\cdot\cos 2\alpha = 2 - 2\sqrt{2}$; $\cos 2\alpha = \frac{2-2\sqrt{2}}{4-2\sqrt{2}}$;

$$\cos 2\alpha = \frac{(1-\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{2+\sqrt{2}-2\sqrt{2}-2}{4-2} = \frac{-\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}.$$

Javob: C

11. (03-2-27). $\sin^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ ni hisoblang.

A) 2 B) $\frac{5}{2}$ C) 4 D) $\frac{3}{2}$ E) $\frac{5}{4}$

Yechilishi:

Keltirish formulalaridan foydalanib, ushbu $\sin\frac{5\pi}{8}=\sin\left(\pi-\frac{3\pi}{8}\right)=\sin\frac{3\pi}{8}$, $\cos\frac{7\pi}{8}=\cos\left(\pi-\frac{\pi}{8}\right)=-\cos\frac{\pi}{8}$ larni berilgan ifodaga qo'yamiz.

$$\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\sin^4\frac{3\pi}{8}+\cos^4\frac{\pi}{8}=\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right)+\left(\cos^4\frac{3\pi}{8}+\sin^4\frac{3\pi}{8}\right)=$$

$$=\left(\sin^4\frac{\pi}{8}+2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}+\cos^4\frac{\pi}{8}-2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)+$$

$$+\left(\cos^4\frac{3\pi}{8}+2\sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8}+\sin^4\frac{3\pi}{8}-2\sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8}\right)=\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)^2-$$

$$-\frac{4\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}}{2}+\left(\sin^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}\right)^2-\frac{4\sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8}}{2}=1-\frac{\left(\sin\frac{\pi}{4}\right)^2}{2}+1-\frac{\left(\sin\frac{3\pi}{4}\right)^2}{2}=$$

$$=1-\frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2}+1-\frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2}=1-\frac{1}{4}+1-\frac{1}{4}=2-\frac{1}{2}=\frac{3}{2}$$

Javob: D

12. (03-7-1). $\cos15^0-\sin15^0=\frac{a}{4\cos15^0}$. a ni toping.

A) $\sqrt{3}$ B) $\sqrt{3}+1$ C) $\sqrt{3}+2$ D) $\sqrt{3}+3$ E) $\sqrt{3}+4$

Yechilishi:

$$\frac{a}{4\cos15^0}=\cos15^0-\sin15^0.$$

$$a=4\cos^2 15^0-4\cos15^0\cdot\sin15^0.$$

Bu yerda $\cos^2 15^0$ uchun $\cos^2 x=\frac{1+\cos2x}{2}$ va $2\cos15^0\sin15^0$ uchun $2\cos x\sin x=\sin2x$ formulalarni qo'llaymiz.

$$a=4\cdot\frac{1+\cos30^0}{2}-2\cdot\sin30^0=2\left(1+\frac{\sqrt{3}}{2}\right)-2\cdot\frac{1}{2}=2+\sqrt{3}-1=\sqrt{3}+1$$

Javob: B

13. (03-4-23). $(tg60^0\cdot\cos15^0-\sin15^0)\cdot7\sqrt{2}$ ning qiymatini toping.

A) 16 B) 12 C) 18 D) 14 E) 10

Yechilishi:

$$(tg60^0\cdot\cos15^0-\sin15^0)\cdot7\sqrt{2}=(\sqrt{3}\cdot\cos15^0-\sin15^0)\cdot7\sqrt{2}=2\left(\frac{\sqrt{3}}{2}\cos15^0-\frac{1}{2}\sin15^0\right)\cdot7\sqrt{2}=$$
$$=2(\cos30^0\cdot\cos15^0-\sin30^0\cdot\sin15^0)\cdot7\sqrt{2}=2\cos(30^0+15^0)\cdot7\sqrt{2}=2\cos45^0\cdot7\sqrt{2}=$$
$$=2\cdot\frac{\sqrt{2}}{2}\cdot7\sqrt{2}=14.$$

Javob: D

14. (05-122-23). $\left(\frac{\sin100^0+\sin20^0}{\sin50^0}\right)^2$ ni hisoblang.

A) $\frac{3}{2}$ B) $\frac{1}{4}$ C) 3 D) 1 E) $\frac{3}{4}$

Yechilishi: $\sin100^0+\sin20^0$ yig'indi uchun $\sin x+\sin y=2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$ formulani qo'llab, soddalashtirib olamiz.

$$\left(\frac{\sin100^0+\sin20^0}{\sin50^0}\right)^2=\left(\frac{2\sin60^0\cdot\cos40^0}{\sin(90^0-40^0)}\right)^2=\left(\frac{2\cdot\frac{\sqrt{3}}{2}\cos40^0}{\cos40^0}\right)^2=(\sqrt{3})^2=3$$

Javob: C

15. (06-109-13). Agar $\cos\alpha=-\frac{1}{7}$ bo'lsa, $\frac{2\sin\alpha+\sin2\alpha}{2\sin\alpha-\sin2\alpha}$ ni hisoblang.

A) $\frac{3}{4}$ B) 0,5 C) $\frac{2}{3}$ D) 3

Yechilishi: $\frac{2\sin\alpha+\sin2\alpha}{2\sin\alpha-\sin2\alpha}$ ni $\cos\alpha$ orqali ifodalab olamiz.

$$\frac{2\sin\alpha+\sin 2\alpha}{2\sin\alpha-\sin 2\alpha}=\frac{2\sin\alpha+2\sin\alpha\cdot\cos\alpha}{2\sin\alpha-2\sin\alpha\cdot\cos\alpha}=\frac{2\sin\alpha(1+\cos\alpha)}{2\sin\alpha(1-\cos\alpha)}=\frac{1+\cos\alpha}{1-\cos\alpha}=$$

$$=\frac{1-\frac{1}{7}}{1+\frac{1}{7}}=\frac{\frac{6}{7}}{\frac{8}{7}}=\frac{6}{8}=\frac{3}{4}$$

Javob: A

16. (06-113-24). Agar $tg\alpha+ctg\alpha=10$ bo‘lsa, $\sin 2\alpha$ ni toping.

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{5}$ D) $\frac{1}{3}$

Yechilishi:

$$tg\alpha+ctg\alpha=10,$$

$$\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}=10.$$

$$\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha\cdot\cos\alpha}=10,$$

$$10\sin\alpha\cdot\cos\alpha=1.$$

$$5\sin 2\alpha=1,\ \sin 2\alpha=\frac{1}{5}$$

Javob: C

17. (98-4-17). Agar $tg\alpha=3$ bo‘lsa, $\frac{3\sin\alpha}{5\sin^3\alpha+10\cos^3\alpha}$ ning qiymati nechaga teng bo‘ladi?

A) $\frac{16}{39}$ B) $\frac{4}{9}$ C) $\frac{8}{15}$ D) $\frac{15}{32}$ E) $\frac{18}{29}$

Yechilishi: $tg\alpha=3$ ekanidan $\frac{\sin\alpha}{\cos\alpha}=3$, $\sin\alpha=3\cos\alpha$ ni topib olamiz. So‘ngra berilgan ifodani $\cos\alpha$ orqali ifodalaymiz.

$$\frac{3\sin\alpha}{5\sin^3\alpha+10\cos^3\alpha}=\frac{3\cdot 3\cos\alpha}{5(3\cos\alpha)^3+10\cos^3\alpha}=\frac{9\cos\alpha}{135\cos^3\alpha+10\cos^3\alpha}=\frac{9\cos\alpha}{145\cos^3\alpha}=$$

$$=\frac{9}{145\cos^2\alpha}=\frac{9}{145}\cdot\frac{1}{\cos^2\alpha}=\frac{9}{145}\cdot\left(tg^2\alpha+1\right)=\frac{9}{145}\cdot 10=\frac{18}{29};$$

(Soddalashtirishda $\frac{1}{\cos^2\alpha}=tg^2\alpha+1$ formuladan foydalandik).

Javob: E

18. (98-8-55). $\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\sin^4\alpha}$ ni soddalashtiring.

A) 3 B) 2 C) 1,5 D) $\frac{1}{3}$ E) 1

Yechilishi:

$$\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\sin^4\alpha}=\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\left(\sin^2\alpha\right)^2}=\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\left(1-\cos^2\alpha\right)^2}=$$

$$=\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+1-2\cos^2\alpha+\cos^4\alpha}=\frac{1+\cos^2\alpha+\cos^4\alpha}{1+\cos^2\alpha+\cos^4\alpha}=1$$

Javob: E

19. (98-8-58). $\frac{1-\sin\alpha-\cos 2\alpha+\sin 3\alpha}{\sin 2\alpha+2\cos\alpha\cdot\cos 2\alpha}$ ni soddalashtiring.

A) $2ctg\alpha$ B) $tg\alpha$ C) $2\sin\alpha$ D) $ctg\alpha$ E) $-ctg\alpha$

Yechilishi:

Ushbu ifodani soddalashtirishda $\sin^2\alpha=\frac{1-\cos 2\alpha}{2}$ va

$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ formulalarni qo'llaymiz.

$$\frac{1-\sin\alpha-\cos2\alpha+\sin3\alpha}{\sin2\alpha+2\cos\alpha\cdot\cos2\alpha}=\frac{(1-\cos2\alpha)+\sin3\alpha-\sin\alpha}{2\sin\alpha\cdot\cos\alpha+2\cos\alpha\cdot\cos2\alpha}=$$

$$=\frac{2\sin^2\alpha+2\cos\frac{3\alpha+\alpha}{2}\sin\frac{3\alpha-\alpha}{2}}{2\cos\alpha(\sin\alpha+\cos2\alpha)}=\frac{2\sin^2\alpha+2\cos2\alpha\sin\alpha}{2\cos\alpha(\sin\alpha+\cos2\alpha)}=$$

$$=\frac{2\sin\alpha(\sin\alpha+\cos2\alpha)}{2\cos\alpha(\sin\alpha+\cos2\alpha)}=tg\alpha$$

Javob: B

20. (99-8-76). $\dfrac{\sin^2 2{,}5\alpha-\sin^2 1{,}5\alpha}{\sin4\alpha\cdot\sin\alpha+\cos3\alpha\cdot\cos2\alpha}$ ni soddalashtiring.

A) $2tg2\alpha$ B) $tg2\alpha\cdot tg\alpha$ C) $2\sin2\alpha$ D) $4\cos^2\alpha$ E) $4\sin^2\alpha$

Yechilishi: Ushbu misolni soddalashtirishning bir necha usullari bo'lib, bu yerda bitta usulini keltiramiz. Bu usulda

$$\sin^2\alpha=\frac{1-\cos2\alpha}{2},\quad \sin\alpha\cdot\sin\beta=\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta)),$$

$$\cos\alpha\cdot\cos\beta=\frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta)),$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\ \text{va}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

formulalardan foydalanamiz.

$$\sin^2 2{,}5\alpha=\frac{1-\cos5\alpha}{2},\quad \sin^2 1{,}5\alpha=\frac{1-\cos3\alpha}{2}$$

$$\sin4\alpha\cdot\sin\alpha=\frac{1}{2}(\cos(4\alpha-\alpha)-\cos(4\alpha+\alpha))$$

$$\sin4\alpha\cdot\sin\alpha=\frac{1}{2}(\cos3\alpha-\cos5\alpha)$$

Xuddi shunday,

$$\cos 3\alpha \cdot \cos 2\alpha = \frac{1}{2}(\cos(3\alpha + 2\alpha) + \cos(3\alpha - 2\alpha)),$$

$$\cos 3\alpha \cdot \cos 2\alpha = \frac{1}{2}(\cos 5\alpha + \cos\alpha)$$

Bu topilganlarni berilgan ifoda o'rniga qo'yamiz.

$$\frac{\frac{1-\cos 5\alpha}{2} - \frac{1-\cos 3\alpha}{2}}{\frac{1}{2}(\cos 3\alpha - \cos 5\alpha) + \frac{1}{2}(\cos 5\alpha + \cos\alpha)} = \frac{1-\cos 5\alpha - 1 + \cos 3\alpha}{\cos 3\alpha - \cos 5\alpha + \cos 5\alpha + \cos\alpha} =$$

$$= \frac{\cos 3\alpha - \cos 5\alpha}{\cos 3\alpha + \cos\alpha}.$$

Bunda yig'indini ko'paytmaga almashtirish formulasini qo'llab, soddalashtirib, natijaga erishamiz

$$\frac{\cos 3\alpha - \cos 5\alpha}{\cos 3\alpha + \cos\alpha} = \frac{2\sin 4\alpha \cdot \sin\alpha}{2\cos 2\alpha \cdot \cos\alpha} = \frac{2\sin 2\alpha \cdot \cos 2\alpha \cdot \sin\alpha}{\cos 2\alpha \cdot \cos\alpha} =$$

$$= \frac{4\sin\alpha \cdot \cos\alpha \cdot \sin\alpha}{\cos\alpha} = 4\sin^2\alpha$$

Javob: E

21. (99-9-32). $\dfrac{\sqrt{3}\cos 2\alpha + \sin 2\alpha}{\cos\alpha + \sqrt{3}\sin\alpha}$ ni soddalashtiring.

A) $2\cos\left(\alpha + \frac{\pi}{3}\right)$ B) $\frac{1}{2}\cos\left(\alpha + \frac{\pi}{6}\right)$ C) $2\cos\left(\alpha - \frac{\pi}{3}\right)$ D) $\frac{1}{2}\sin\left(\alpha + \frac{\pi}{6}\right)$

E) $2\cos\left(\alpha + \frac{\pi}{6}\right)$

Yechilishi: $\dfrac{\sqrt{3}\cos 2\alpha + \sin 2\alpha}{\cos\alpha + \sqrt{3}\sin\alpha} = \dfrac{\frac{\sqrt{3}}{2}\cos 2\alpha + \frac{1}{2}\sin 2\alpha}{\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha}.$

Kasrning surat va maxrajini 2ga bo'ldik.

$\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}$, $\frac{1}{2}=\sin\frac{\pi}{6}$, $\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$, $\frac{1}{2}=\cos\frac{\pi}{3}$ lardan

foydalanib, $\dfrac{\sin\frac{\pi}{3}\cos 2\alpha+\cos\frac{\pi}{3}\sin 2\alpha}{\sin\frac{\pi}{6}\cos\alpha+\cos\frac{\pi}{6}\sin\alpha}$ ko'rinishga keltiramiz. Endi surat va maxrajda $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ formulani qo'llash mumkin.

Demak,

$$\frac{\sin\left(\frac{\pi}{3}+2\alpha\right)}{\sin\left(\frac{\pi}{6}+\alpha\right)}=\frac{\sin\left(2\left(\frac{\pi}{2}+\alpha\right)\right)}{\sin\left(\frac{\pi}{6}+\alpha\right)}=\frac{2\sin\left(\frac{\pi}{6}+\alpha\right)\cos\left(\frac{\pi}{6}+\alpha\right)}{\sin\left(\frac{\pi}{6}+\alpha\right)}=2\cos\left(\frac{\pi}{6}+\alpha\right)$$

Javob: E

22.(03-3-41). $\dfrac{\sin(\pi+\alpha)}{\sin\left(\frac{3\pi}{2}+\alpha\right)}+\dfrac{\cos(\pi-\alpha)}{\cos\left(\frac{\pi}{2}+\alpha\right)-1}$ ni soddalashtiring.

A) $\frac{1}{\cos\alpha}$ B) $\frac{1}{\sin\alpha}$ C) $\sin\alpha$ D) $\cos\alpha$ E) 1

Yechilishi:

$$\frac{\sin(\pi+\alpha)}{\sin\left(\frac{3\pi}{2}+\alpha\right)}+\frac{\cos(\pi-\alpha)}{\cos\left(\frac{\pi}{2}+\alpha\right)-1}=\frac{-\sin\alpha}{-\cos\alpha}+\frac{-\cos\alpha}{-\sin\alpha-1}=\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{1+\sin\alpha}=$$

$$=\frac{\sin^2\alpha+\cos^2\alpha+\sin\alpha}{\cos\alpha(1+\sin\alpha)}=\frac{1+\sin\alpha}{\cos\alpha(1+\sin\alpha)}=\frac{1}{\cos\alpha}$$

Javob: A.

23.(03-9-27). $\dfrac{1-\sin^2\dfrac{\alpha}{8}-\cos^2\alpha-\sin^2\alpha}{4\sin^4\dfrac{\alpha}{16}}$ ni soddalashtiring.

A) $tg^2\dfrac{\alpha}{16}$ B) 1 C) -1 D) $ctg^2\dfrac{\alpha}{16}$ E) $-ctg^2\dfrac{\alpha}{16}$

Yechilishi:

$$\frac{1-\sin^2\frac{\alpha}{8}-\cos^2\alpha-\sin^2\alpha}{4\sin^4\frac{\alpha}{16}}=\frac{1-\sin^2\frac{\alpha}{8}-\left(\cos^2\alpha+\sin^2\alpha\right)}{4\sin^4\frac{\alpha}{16}}=\frac{1-\sin^2\frac{\alpha}{8}-1}{4\sin^4\frac{\alpha}{16}}=$$

$$=\frac{-4\sin^2\frac{\alpha}{16}\cdot\cos^2\frac{\alpha}{16}}{4\sin^4\frac{\alpha}{16}}=-\frac{\cos^2\frac{\alpha}{16}}{\sin^2\frac{\alpha}{16}}=-ctg^2\frac{\alpha}{16}$$

Javob: E.

24.(05-101-23). $\dfrac{\sin\alpha+\sin 2\alpha-\sin(\pi+3\alpha)}{2\cos\alpha+1}$ ni soddalashtiring.

A) $\sin 2\alpha$ B) $1+\sin\alpha$ C) $\sin\alpha$ D) $\cos 2\alpha$ E) $\cos\alpha$

Yechilishi: Ifodani soddalashtirishda

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \quad \text{va} \quad \sin(\pi+\alpha)=-\sin\alpha$$

formulalardan foydalanamiz

$$\frac{\sin\alpha+\sin 2\alpha-\sin(\pi+3\alpha)}{2\cos\alpha+1}=\frac{\sin\alpha+\sin 2\alpha+\sin 3\alpha}{2\cos\alpha+1}=$$

$$=\frac{(\sin\alpha+\sin 3\alpha)+\sin 2\alpha}{2\cos\alpha+1}=\frac{2\sin\frac{\alpha+3\alpha}{2}\cos\frac{\alpha-3\alpha}{2}+\sin 2\alpha}{2\cos\alpha+1}=$$

$$=\frac{2\sin 2\alpha\cdot\cos\alpha+\sin 2\alpha}{2\cos\alpha+1}=\frac{\sin 2\alpha(2\cos\alpha+1)}{2\cos\alpha+1}=\sin 2\alpha$$

Javob: A.

25.(05-104-23). $\dfrac{\cos\alpha-2\sin3\alpha-\cos5\alpha}{\sin5\alpha-2\cos3\alpha-\sin\alpha}$ ni soddalashtiring.

A) $tg3\alpha$ B) 2 C) 1 D) $ctg\alpha$ E) $tg\alpha$

Yechilishi: Ifodani soddalashtirishda

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2},$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

formulalarni qo'llaymiz

$$\frac{\cos\alpha-2\sin3\alpha-\cos5\alpha}{\sin5\alpha-2\cos3\alpha-\sin\alpha}=\frac{(\cos\alpha-\cos5\alpha)-2\sin3\alpha}{(\sin5\alpha-\sin\alpha)-2\cos3\alpha}=$$

$$=\frac{-2\sin\frac{\alpha+5\alpha}{2}\cdot\sin\frac{\alpha-5\alpha}{2}-2\sin3\alpha}{2\sin\frac{5\alpha-\alpha}{2}\cdot\cos\frac{5\alpha+\alpha}{2}-2\cos3\alpha}=\frac{2\sin3\alpha\sin2\alpha-2\sin3\alpha}{2\sin2\alpha\cdot\cos3\alpha-2\cos3\alpha}=$$

$$=\frac{2\sin3\alpha(\sin2\alpha-1)}{2\cos3\alpha(\sin2\alpha-1)}=tg3\alpha$$

Javob: A.

26.(05-106-23). $\dfrac{\sin\alpha+\cos\alpha}{\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right)}$ ni soddalashtiring.

A) 1,5 B) $tg\left(\frac{\pi}{4}+\alpha\right)$ C) 1,6 D) 1 E) $ctg\left(\frac{\pi}{4}+\alpha\right)$

Yechilishi: Ushbu ifodani soddalashtirish uchun kasrning suratida quyidagicha almashtirishni bajaramiz

$$\sin\alpha+\cos\alpha=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\alpha+\frac{1}{\sqrt{2}}\cos\alpha\right)=\sqrt{2}\left(\cos\frac{\pi}{4}\cos\alpha+\sin\frac{\pi}{4}\cdot\sin\alpha\right)=\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right)$$

Bu ifodani o'rniga qo'yib, $\frac{\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right)}{\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right)}=1$ natijaga erishamiz.

Javob: D.

27.(05-111-23). $\frac{\cos^4\alpha-\cos^2\alpha+\sin^2\alpha}{\sin^4\alpha-\sin^2\alpha+\cos^2\alpha}$ ni soddalashtiring.

A) $ctg^4\alpha$ B) $2ctg^4\alpha$ C) $tg^4\alpha$ D) $\frac{1}{2}tg^2\alpha$ E) $tg^2\alpha$

Yechilishi:

$$\frac{\cos^4\alpha-\cos^2\alpha+\sin^2\alpha}{\sin^4\alpha-\sin^2\alpha+\cos^2\alpha}=\frac{-\cos^2\alpha(1-\cos^2\alpha)+\sin^2\alpha}{-\sin^2\alpha(1-\sin^2\alpha)+\cos^2\alpha}=\frac{-\cos^2\alpha\cdot\sin^2\alpha+\sin^2\alpha}{-\sin^2\alpha\cdot\cos^2\alpha+\cos^2\alpha}=$$

$$=\frac{\sin^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha(1-\sin^2\alpha)}=\frac{\sin^2\alpha\cdot\sin^2\alpha}{\cos^2\alpha\cdot\cos^2\alpha}=\frac{\sin^4\alpha}{\cos^4\alpha}=tg^4\alpha$$

Javob: C.

28.(05-119-23). $\frac{1-\sin^4\alpha-\cos^4\alpha}{\sin^4\alpha}$ ni soddalashtiring.

A)2 B) $2ctg^2\alpha$ C) $2tg^2\alpha$ D) $\sin^2\alpha$ E) $\frac{1}{\cos^2\alpha}$

Yechilishi:

$$\frac{1-\sin^4\alpha-\cos^4\alpha}{\sin^4\alpha}=\frac{(\sin^2\alpha+\cos^2\alpha)^2-\sin^4\alpha-\cos^4\alpha}{\sin^4\alpha}=$$

$$=\frac{\sin^4\alpha+2\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha-\sin^4\alpha-\cos^4\alpha}{\sin^4\alpha}=\frac{2\sin^2\alpha\cdot\cos^2\alpha}{\sin^4\alpha}=2ctg^2\alpha$$

Javob: B.

(Soddalashtirish jarayonida $\sin^2\alpha + \cos^2\alpha = 1$ formuladan foydalandik).

29.(05-132-23). $\sin^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$ ni soddalashtiring.

A) 1 B) $\frac{1}{4}$ C) 2 D) $2\sqrt{2}$ E) 4

Yechilishi:

$\sin^2\frac{5\pi}{8}$ va $\cos^2\frac{7\pi}{8}$ larni quyidagicha shakl almashtirib olamiz.

$$\sin^2\frac{5\pi}{8} = \left(\sin\left(\pi - \frac{3\pi}{8}\right)\right)^2 = \sin^2\frac{3\pi}{8}, \quad \left(чунки \ \pi - \frac{3\pi}{8} = \frac{5\pi}{8}\right)$$

$$\cos^2\frac{7\pi}{8} = \left(\cos\left(\pi - \frac{\pi}{8}\right)\right)^2 = \cos^2\frac{\pi}{8}, \quad \left(чунки \ \pi - \frac{\pi}{8} = \frac{7\pi}{8}\right)$$

Endi topilgan ifodalarni berilgan ifodaga qo'yamiz.

$$\sin^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{8} =$$

$$= \left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right) + \left(\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8}\right) = 1 + 1 = 2$$

Javob: C.

30.(06-101-13). $\frac{2}{tg2\alpha - ctg2\alpha}$ ni soddalashtiring.

A) $-2tg4\alpha$ B) $\cos 4\alpha$ C) $-tg4\alpha$ D) $tg4\alpha$

Yechilishi:

$$\frac{2}{tg2\alpha - ctg2\alpha} = \frac{2}{tg2\alpha - \frac{1}{tg2\alpha}} = \frac{2tg2\alpha}{tg^2 2\alpha - 1} = -\frac{2tg2\alpha}{1 - tg^2 2\alpha} = -tg4\alpha$$

(Bu yerda $tg\alpha = \frac{2tg\left(\frac{\alpha}{2}\right)}{1 - tg^2\left(\frac{\alpha}{2}\right)}$ formulani $tg4\alpha$ uchun qo'lladik. Bu misolni $tg2\alpha$ va $ctg2\alpha$ ni $\sin 2\alpha$ va $\cos 2\alpha$ orqali ifodalab ham soddalashtirsa bo'ladi).

Javob: C.

31.(06-102-13). $\frac{1 - \cos 4\alpha + \sin^2 2\alpha}{3\cos^2 2\alpha}$ ni soddalashtiring.

A) $3tg^2 2\alpha$ B) $3ctg^2 2\alpha$ C) $tg^2 2\alpha$ D) $1{,}5ctg^2 2\alpha$

Yechilishi:

$$\frac{1 - \cos 4\alpha + \sin^2 2\alpha}{3\cos^2 2\alpha} = \frac{2\sin^2 2\alpha + \sin^2 2\alpha}{3\cos^2 2\alpha} = \frac{3\sin^2 2\alpha}{3\cos^2 2\alpha} = tg^2 2\alpha$$

(Bu yerda $\sin^2 2\alpha = \frac{1 - \cos 4\alpha}{2}$ formuladan foydalandik).

Javob: C.

32.(06-110-13). $1 + \frac{\sin^4\alpha + \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha}$ ni soddalashtiring.

A) $1 - tg^2\alpha$ B) $tg^2\alpha$ C) $1 - ctg^2\alpha$ D) $\frac{1}{\cos^2\alpha}$

Yechilishi:

$$1 + \frac{\sin^4\alpha + \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} = \frac{\cos^2\alpha + (1 - \cos^2\alpha)^2 + \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} =$$

$$= \frac{\cos^2\alpha + 1 - 2\cos^2\alpha + \cos^4\alpha + \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} = \frac{1 - \cos^2\alpha + \cos^4\alpha + \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} =$$

$$= \frac{1 - \cos^2\alpha(1 - \sin^2\alpha) + \cos^4\alpha}{\cos^2\alpha} = \frac{1 - \cos^4\alpha + \cos^4\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$$

Javob: D.

33.(06-114-13). $\dfrac{1+\cos 2\alpha+\cos 4\alpha+\cos 6\alpha}{\sin 4\alpha+2\sin 2\alpha\cdot\cos 4\alpha}$ ni soddalashtiring.

A) $tg\alpha$ B) $2ctg 2\alpha$ C) $ctg 2\alpha$ D) $2\sin 2\alpha$

Yechilishi:

Berilgan ifodani soddalashtirishda

$\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$ formulani $\cos 4\alpha+\cos 6\alpha$

yig'indi uchun qo'llaymiz va $1+\cos 2\alpha=2\cos^2\alpha$ formuladan foydalanib, ifodani quyidagi ko'rinishga keltiramiz.

$$\frac{2\cos^2\alpha+2\cos 5\alpha\cdot\cos\alpha}{2\sin 2\alpha\cdot\cos 2\alpha+2\sin 2\alpha\cdot\cos 4\alpha}=\frac{2\cos\alpha(\cos\alpha+\cos 5\alpha)}{2\sin 2\alpha(\cos 2\alpha+\cos 4\alpha)}$$

Yig'indini ko'paytmaga almashtirish formulasini surat va maxrajdagi qavs ichidagi yig'indi uchun yana bir karra qo'llaymiz, ya'ni

$$\frac{\cos\alpha\cdot 2\cos 3\alpha\cdot\cos 2\alpha}{\sin 2\alpha\cdot 2\cos 3\alpha\cdot\cos\alpha}=\frac{\cos 2\alpha}{\sin 2\alpha}=ctg 2\alpha$$

Javob: C.

34.(06-122-13). $\dfrac{\sin 4\alpha+2\cos 2\alpha\cdot\cos 4\alpha}{1-\sin 2\alpha-\cos 4\alpha+\sin 6\alpha}$ ni soddalashtiring.

A) $2\sin 2\alpha$ B) $2tg 2\alpha$ C) $ctg 2\alpha$ D) $4tg 2\alpha$

Yechilishi:

$$\frac{2\sin 2\alpha\cdot\cos 2\alpha+2\cos 2\alpha\cdot\cos 4\alpha}{1-\cos 4\alpha-(\sin 2\alpha-\sin 6\alpha)}=\frac{2\cos 2\alpha(\sin 2\alpha+\cos 4\alpha)}{2\sin^2 2\alpha-2\sin\dfrac{2\alpha-6\alpha}{2}\cos\dfrac{2\alpha+6\alpha}{2}}=$$

$$=\frac{2\cos 2\alpha(\sin 2\alpha+\cos 4\alpha)}{2\sin^2 2\alpha+2\sin 2\alpha\cos 4\alpha}=\frac{2\cos 2\alpha(\sin 2\alpha+\cos 4\alpha)}{2\sin 2\alpha(\sin 2\alpha+\cos 4\alpha)}=ctg 2\alpha$$

(Berilgan ifodani soddalashtirishda $\sin 2\alpha = 2\sin\cdot\cos\alpha$ formulani $\sin 4\alpha$ uchun, $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$ formulani $1-\cos 4\alpha$ uchun (ya'ni $\sin^2 2\alpha = \frac{1-\cos 4\alpha}{2}$ ko'rinishda) qo'lladik. $\sin 2\alpha - \sin 6\alpha$ uchun esa $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$ formuladan foydalanib, ko'paytmaga keltirdik).

Javob: C.

TRIGONOMETRIK TENGLAMALAR, ULARNING TURLARI VA YECHISH USULLARI

Eng sodda trigonometrik tenglamalarga

$\sin x = a$, bu yerda $|a| \leq 1$, (1)

$\cos x = a$, bu yerda $|a| \leq 1$, (2)

$tgx = a$ (3) va $ctgx = a$ (4) ko'rinishdagi tenglamalar kiradi.

(1) tenglamaning yechimi $x = (-1)^k \arcsin a + \pi k, \quad k \in Z$,

(2) tenglamaning yechimi $x = \pm \arccos a + 2\pi k, \quad k \in Z$,

(3) tenglamaning yechimi $x = arctga + \pi k, \quad k \in Z$,

(4) tenglamaning yechimi $x = arcctga + \pi k, \quad k \in Z$ ko'rinishda izlanadi.

Eng sodda trigonometrik tenglamalarni yuqoridagi umumiy formulalardan foydalanmay yozish mumkin.

Jumladan, quyida yechimlari bilan berilayotgan tenglamalar ana shunday tenglamalardir:

$\sin x = 0, \quad x = \pi k, \quad k \in Z;$

$\sin x = 1, \quad x = \frac{\pi}{2} + 2\pi k, \quad k \in Z;$

$\sin x = -1, \quad x = -\frac{\pi}{2} + 2\pi k, \quad k \in Z;$

$\cos x = 0, \quad x = \frac{\pi}{2} + \pi k, \quad k \in Z;$

$\cos x = 1, \quad x = 2\pi k, \quad k \in Z;$

$\cos x = -1, \quad x = \pi + 2\pi k, \quad k \in Z;$

$tgx = 0, \quad x = \pi k, \quad k \in Z$

Trigonometrik tenglamalarni yechgandan keyin quyidagi hollarda albatta javobni tekshirib ko'rish shart:

1) Agar tenglamani yechish jarayonida shakl almashtirishlar natijasida tenglamaning aniqlanish sohasi kengayib ketsa;

2) Agar tenglamani yechish jarayonida tenglikning ikkala tomoni bir xil darajaga ko'tarilgan bo'lsa;

3) Agar tenglamani yechish jarayonida foydalanilgan trigonometrik ayniyatlarning tenglikdan o'ng va chap tomonlari turli aniqlanish sohasiga ega bo'lsa.

Jumladan,

$$\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}=\sin\alpha,$$

$$\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}=\cos\alpha,$$

$$\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}=tg\alpha$$

$$tg\,\alpha\;ctg\,\alpha=1,$$

$$\frac{1-\cos\alpha}{\sin\alpha}=tg\frac{\alpha}{2},$$

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\,tg\beta}$$

va hokazo, ayniyatlarni "chapdan o'ngga" qo'llash tenglamaning aniqlanish sohasini kengayib ketishiga olib keladi. Natijada begona ildizlar paydo bo'lib qoladi. Ammo yuqoridagi ayniyatlarni "o'ngdan chapga" qo'llash tenglamaning aniqlanish sohasini torayishiga olib keladi. Bu holda ildizlar yo'qolib ketishi mumkin.

Ayrim trigonometrik formulalarda tenglikning ikkala tomoni ham bir xil aniqlanish sohasiga ega bo'lish bilan birga o'zgaruvchining barcha qiymatlari uchun o'rinli bo'lmasligi mumkin.

Masalan,

$$tg\alpha=\frac{\sin 2\alpha}{1+\cos 2\alpha}$$

formulani olaylik. Ushbu formulada tenglikning ikkala tomoni ham bir xil aniqlanish sohasiga ega bo'lib, tenglik faqat $\alpha\neq\frac{\pi}{2}+2\pi n,\ n\in Z$ qiymatlar uchun o'rinlidir.

Ko'pgina trigonometrik formulalar α o'zgaruvchining barcha qiymatlari uchun o'rinli bo'ladi.

Masalan,

$$\sin 2\alpha = 2\sin\alpha\,\cos\alpha,\;\; \cos 2\alpha = \cos^2\alpha - \sin^2\alpha\,,$$

$$\sin^2\frac{\alpha}{2} = \frac{1}{2}(1-\cos\alpha,),$$

$$\cos^2\frac{\alpha}{2} = \frac{1}{2}(1+\cos\alpha),$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\,\sin\beta\,,$$
$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$$

va sinuslar ko'paytmasini, kosinuslar ko'paytmasini, sinuslar yig'indisi va ayirmasini topish formulalari ham ana shunday formulalardir.

BIR JINSLI TENGLAMALARNI YECHISH

1-darajali bir jinsli tenglamaga

$$a\sin x + b\cos x = 0$$

ko'rinishidagi tenglamalar kiradi.

2-darajali bir jinsli tenglamalar esa

$$a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0$$

ko'rinishida bo'ladi.

$$a\sin x + b\cos x = 0$$

ko'rinishidagi bir jinsli tenglamalarni yechishda quyidagicha mulohaza yuritiladi.

Agar $a \neq 0$ bo'lsa, berilgan bir jinsli tenglamani $\cos x = 0$ tenglamaning ildizlari qanoatlantirmaydi. Shuning uchun $a \neq 0$ bo'lgan holda tenglikning ikkala tomonini $\cos x$ ga bo'lish bilan berilgan tenglamaga teng kuchli tenglamaga keltirib yechiladi. Xuddi shunday $a \neq 0$ bo'lganda 2-darajali bir jinsli tenglamalarni yechish uchun ham tenglikning ikkala tomoni $\cos^2 x$ ga bo'linadi. Shunday qilib, 1-darajali bir jinsli tenglama

$$atgx = 0$$

ko'rinishdagi va 2-darajali bir jinsli tenglama

$$atg^2 x + btgx + c = 0$$

ko'rinishidagi tenglamaga keltirib yechiladi.

Misol: $\sin^2 x - 3\sin x\cos x + 2\cos^2 x = 0$ tenglamani yeching.

Yechish: Berilgan tenglama 2-darajali bir jinsli tenglama bo'lgani uchun tenglikning ikkala tomonini $\cos^2 x$ ga bo'lsak bo'ladi (chunki $a = 1$). U holda ushbu

$$tg^2 x - 3tgx + 2 = 0$$

tenglamani hosil qilamiz. Bu tenglamada $tgx = y$ deb belgilash kiritib, $y^2 - 3y + 2 = 0$ kvadrat tenglamani yechib, uning ildizlari $y_1=1$ va $y_2 = 2$ ekanini topamiz. Bundan

$tgx = 1, \quad x = \frac{\pi}{4} + \pi n, \quad n \in Z$ va

$tgx = 2, \quad x = arctg 2 + \pi n, \quad n \in Z$ javobga ega bo'lamiz.

Ayrim bir jinsli bo'lmagan tenglamalarni ham bir jinsli tenglama ko'rinishiga keltirib yechish mumkin.

Misol. $2\sin^3 x = \cos x$ tenglamani yeching.

Yechish: $\cos x = \cos x(\cos^2 x + \sin^2 x)$ tenglik har doim o'rinli ($\cos^2 x + \sin^2 x = 1$ formuladan foydalanamiz). U holda berilgan tenglama

$$2\sin^3 x = \cos x \sin^2 x + \cos^3 x$$

tenglamaga teng kuchlidir. Bu tenglama $\sin x$ va $\cos x$ ga nisbatan bir jinsli tenglamadir. Agar $\cos x = 0$ bo'lsa, tenglama yechimga ega bo'lmaydi. U holda $\cos x \neq 0$ deb, tenglikning ikkala tomonini $\cos^3 x$ ga bo'lib,

$$2tg^3 x - tg^2 x - 1 = 0$$

tenglama hosil qilamiz. Bu yerda ham $tgx = y$ deb belgilab, hosil bo'lgan

$$2y^3 - y^2 - 1 = 0$$

tenglamada tenglikning chap tomonini kopaytuvchilarga ajratsak,

$$(y-1)(2y^2 + y + 1) = 0$$

tenglama hosil bo'ladi. Bundan $y = 1$, $tgx = 1$, $x = \frac{\pi}{4} + \pi k$, $k \in Z$ yagona ildizni topamiz. Ikkinchi qavs ichidagi ifoda esa y ning har qanday qiymatida $2y^2 + y + 1 > 0$ shartni qanoatlantiradi.

TRIGONOMETRIK TENGLAMALARNI $t = tg\,\frac{x}{2}$ BELGILASHDAN FOYDALANIB YECHISH

Argumentida sin*x*, cos*x*, *tgx* va *ctgx* funksiyalar qatnashgan tenglamani shu funksiyalarga nisbatan ratsional tenglamaga keltirib yechishda $t = tg\,\frac{x}{2}$ belgilashdan foydalanish juda qulaylik tug'diradi.

Buning uchun ushbu

$$\sin x = \frac{2tg\,\frac{x}{2}}{1 + tg^2\,\frac{x}{2}}, \quad \cos x = \frac{1 - tg^2\,\frac{x}{2}}{1 + tg^2\,\frac{x}{2}},$$

$$tgx = \frac{2tg\,\frac{x}{2}}{1 - tg^2\,\frac{x}{2}}, \quad ctgx = \frac{1 - tg^2\,\frac{x}{2}}{2tg\,\frac{x}{2}}$$

formulalarni bilish zarur.

Misol. $\sin x + ctg\,\frac{x}{2} = 2$ tenglamani yeching.

Tenglamani yechish uchun $\sin x = \frac{2tg\,\frac{x}{2}}{1 + tg^2\,\frac{x}{2}}$ va $ctgx = \frac{1}{tgx}$ formulalardan foydalanib, $\frac{2tg\,\frac{x}{2}}{1 + tg^2\,\frac{x}{2}} + \frac{1}{tg\,\frac{x}{2}} = 2$ ko'rinishda yozib olamiz.

So'ngra $tg\,\frac{x}{2} = t$ belgilashni kiritib, $\frac{2t}{1 + t^2} + \frac{1}{t} = 2$, $2t^3 - 3t^2 + 2t - 1 = 0$, tenglamani yechamiz.

$2t^3 - 3t^2 + 2t - 1 = (t - 1)(2t^2 - t + 1)$ bo'lgani uchun $(t - 1)(2t^2 - t + 1) = 0$, bu yerda $2t^2 - t + 1 \neq 0$. $t \in R$.

Demak, $t-1=0$, $t=1$, $tg\frac{x}{2}=1$.

$\frac{x}{2}=\frac{\pi}{4}+\pi k$, $k\in Z$, $x=\frac{\pi}{2}+2\pi k$, $k\in Z$, javobni hosil qilamiz.

Trigonometrik tenglamalarni yechishda $t=tg\frac{x}{2}$ deb belgilab olishdan foydalanish usuli o'rniga qo'yishning universal usuli bo'lib, $x\neq\pi+2\pi k,\ k\in Z$ bo'lganda o'rinli bo'ladi. Bunda, $x=\pi+2\pi k,\ k\in Z$ qiymatlar berilgan tenglamaning ildizlari bo'lib qolishi ham mumkin, ana shundan ehtiyot bo'lish kerak.

$$\sin ax+\sin bx=0,\quad \sin ax-\sin bx=0$$

$$\cos ax+\cos bx=0,\quad \cos ax-\cos bx=0$$

KO'RINISHDAGI TENGLAMALARNI YECHISH

Bunday ko'rinishdagi tenglamalarni yechishda

$$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2},$$

$$\sin x-\sin y=2\sin\frac{x-y}{2}\cos\frac{x+y}{2},$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2},$$

$$\cos x-\cos y=2\sin\frac{y-x}{2}\sin\frac{y+x}{2}.$$

formulalardan foydalanib, qulay holga keltiriladi.

Misol. 1. $\sin 2x+\sin 4x=0$ tenglamani yeching.

Yechish: Yuqoridagi birinchi formulaga ko'ra,

$$2\sin 3x\cos x=0,\Rightarrow\begin{cases}\sin 3x=0,\\ \cos x=0;\end{cases}\Rightarrow\left[\begin{array}{l}x=\dfrac{\pi k}{3},\ k\in Z\\ x=\dfrac{\pi}{2}+\pi k,\ k\in Z.\end{array}\right.$$

Agar trigonometrik tenglamalar $\sin ax+\cos bx=0$ yoki $\sin ax-\cos bx=0$ ko'rinishda bo'lsa, berilgan tenglamani keltirish formulalardan foydalanib, bir nomdagi trigonometrik funksiyalar qatnashgan tenglamalarga keltirilib, yuqoridagi formulalardan foydalanib yechiladi.

2. (01-8-53) Ushbu $\sin 3x+\sin 5x=\sin 4x$ tenglamaning nechta ildizi $|x|\le\frac{\pi}{2}$ tengsizlikni qanoatlantiradi?

A)2 B)3 C)4 D)5 E)7

Yechlishi: Tenglamani yechishda $\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$ formuladan foydalanib, tenglikning chap tomonini ko'paytmaga almashtirib olamiz.

$\sin 3x + \sin 5x = 2\sin 4x \cdot \cos x$. Buni o'rniga qo'yib, tenglamani yechamiz.

$2\sin 4x \cdot \cos x = \sin 4x$, $\quad \sin 4x(2\cos x - 1) = 0$,

$$\begin{cases} \sin 4x = 0, \\ 2\cos x = 1, \end{cases} \Rightarrow \begin{cases} 4x = n\pi, \\ \cos x = \dfrac{1}{2}, \end{cases} \Rightarrow \begin{cases} x = \dfrac{n\pi}{4}, \quad n \in Z; \\ x = \pm\dfrac{\pi}{3} + 2\pi k, \quad k \in Z. \end{cases}$$

Bundan ko'rinib turibdiki, $-\dfrac{\pi}{2}$, $-\dfrac{\pi}{4}$, 0, $\dfrac{\pi}{4}$, $\dfrac{\pi}{2}$, $\dfrac{\pi}{3}$, $-\dfrac{\pi}{3}$ lar berilgan shartni qanoatlantiradi.

Javob:E

KO'PAYTUVCHILARGA AJRATIB YECHISH

Ayrim trigonometrik tenglamalarni yechishda ko'paytuvchilarga ajratib yechish usulidan foydalanish mumkin. Buni misol asosida tushuntiramiz.

Misol.

Yechilishi: $5\sin^2 x+\sqrt{3}\sin x\cos x+6\cos^2 x=5$ tenglamani yeching.

Berilgan tenglamani $\sin^2 x+\cos^2 x=1$ formuladan foydalanib bir jinsli tenglama ko'rinishiga keltiramiz.

$$5\sin^2 x+\sqrt{3}\sin x\cos x+6\cos^2 x=5(\sin^2 x+\cos^2 x),$$

$$5\sin^2 x+\sqrt{3}\sin x\cos x+6\cos^2 x-5\sin^2 x-5\cos^2 x=0,$$

$$\sqrt{3}\sin x\cos x+\cos^2 x=0$$

Bu tenglamani $\cos^2 x$ ga bo'lib yuborish mumkin emas, chunki bu hol ildizlarning yo'qolib ketishiga olib keladi. Shuning uchun oxirgi tenglamani ko'paytuvchilarga ajratamiz.

$$\cos x(\sqrt{3}\sin x+\cos x)=0,$$

$$\begin{cases}\cos x=0,\\ \sqrt{3}\sin x+\cos x=0,\end{cases}$$

sistemadagi birinchi tenglamadan $x=\frac{\pi}{2}+\pi k,\ k\in Z$ yechimni topamiz.

Ikkinchi tenglamada esa tenglikning ikkala tomonini $\cos x$ ga bo'lib (bu holda bo'lish mumkin),

$$\sqrt{3}tgx=-1,$$

$$tgx=-\frac{1}{\sqrt{3}}$$

tenglamani hosil qilamiz. Bu tenglamaning yechimi $x=-\frac{\pi}{6}+\pi n,\ \ n\in Z$

dan iborat bo'lib, javob $x=\frac{\pi}{2}+\pi k,\ \ k\in Z$, $\quad x=-\frac{\pi}{6}+\pi n,\ \ n\in Z$ bo'ladi.

2) (96-1-60). Agar $90^0<x<180^0$ bo'lsa, $\cos 2x\cdot\sin x=\cos 2x$ tenglamaning ildizlarini toping.

A) 120^0 B) 110^0 C) 170^0 D) 135^0 E) 135^0 va 165^0

Yechilishi: $\cos 2x(\sin x - 1) = 0)$

1) $\cos 2x = 0$

$2x = 90^0 + 180^0 n,$

$x = 45^0 + 90^0 n,\ n \in Z$

n	0	1
x	45^0	135^0

2) $\sin x - 1 = 0$

$\sin x = 1$

$x = 90^0 + 360^0 k,\ \ k \in Z$

Javob: D.

TRIGONOMETRIK TENGLAMALARNI YORDAMCHI BURCHAK KIRITISH USULI BILAN YECHISH

Ba'zi trigonometrik tenglamalar $a\cos x + b\sin x = c$ ko'rinishda berilib, bunday tenglamalar

$$\sin(x+\varphi) = \frac{c}{\sqrt{a^2+b^2}}$$

tenglamaga teng kuchli bo'lib, bu yerda φ burchak $\sin\varphi = \frac{b}{\sqrt{a^2+b^2}}$ va

$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}$ formulalar orqali topiladi.

a va b ning ishoralariga qarab, φ burchakni $arctg\frac{b}{a}$ (agar $a>0$, $b>0$ yoki $a>0$, $b<0$ bo'lsa) ko'rinishida yoki $\pi + arctg\frac{b}{a}$ (yuqoridagi qavs ichidagi shartdan boshqa hollarda $\varphi = \pi + arctg\frac{b}{a}$) ko'rinishda olish mumkin.

Misol. Trigonometrik tenglamalarni yordamchi burchak kiritish usulidan foydalanib yeching.

1) $\sqrt{3}\sin x - \cos x - 1 = 0$

$$\sin x\cdot\frac{\sqrt{3}}{2} - \cos x\cdot\frac{1}{2} = \frac{1}{2}$$

$$\sin x\cdot\cos\frac{\pi}{6} - \cos x\cdot\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$$

$$x-\frac{\pi}{6} = (-1)^k \arcsin\frac{1}{2} + \pi k$$

$$x = \frac{\pi}{6} + \pi k + (-1)^k\frac{\pi}{6},\quad k\in Z$$

2) $\sin x + \sqrt{3}\cos x = 1$

$$\sin x\cdot\frac{1}{2} + \cos x\cdot\frac{\sqrt{3}}{2} = \frac{1}{2}$$

$$\sin x\cdot\cos\frac{\pi}{3} + \cos x\cdot\sin\frac{\pi}{3} = \frac{1}{2}$$

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$$

$$x+\frac{\pi}{3} = (-1)^k \arcsin\frac{1}{2} + \pi k$$

$$x = \pi k - \frac{\pi}{3} + (-1)^k\frac{\pi}{6},\quad k\in Z$$

BAHOLASH USULIDAN FOYDALANIB YECHISH

Ayrim trigonometrik tenglamalarni yechishda $-1 \le \sin x \le 1$, $-1 \le \cos x \le 1$ formulalardan foydalanib, maqsadga erishish mumkin.

Misol. 1. $\sin 3x + \cos 2x + 2 = 0$ tenglamani yeching.

Yechish: $-1 \le \sin 3x \le 1$ va $-1 \le \cos 2x \le 1$ ekanidan

$\sin 3x + \cos 2x = -2$ bo'ladi.

Bu yerda tenglik faqat $\sin 3x = -1$ va $\cos 2x = -1$ bo'lganda o'rinlidir. Demak,

$$\sin 3x = -1 \text{ dan } x = -\frac{\pi}{6} + \frac{2\pi k}{3},\ k \in Z$$

$$\cos 2x = -1 \text{ dan } x = \frac{\pi}{2} + \pi n,\quad n \in Z.$$

Bu ikkala javobni umumlashtirib,

$x = \frac{\pi}{2} + 2\pi m,\quad m \in Z$ ko'rinishdagi javobni olamiz.

Misol. 2. $\sin x + \sin 3x = 1$ tenglamani yeching.

Yechish: $|\sin x| \le 1$ va $|\sin 3x| \le 1$ bo'lganidan berilgan tenglama

$$\begin{cases} \sin x = 1, \\ \sin 3x = 1 \end{cases}$$

sistemaga teng kuchli.

Birinchi tenglamaning yechimi: $x = \frac{\pi}{2} + 2\pi k,\quad k \in Z$.

Ikkinchi tenglamaniki: $x = \frac{\pi}{6} + \frac{2\pi n}{3},\ n \in Z$.

U holda berilgan tenglamaning yechimi:

$\frac{\pi}{2} + 2\pi k = \frac{\pi}{6} + \frac{2\pi n}{3},\ \ 2n - 6k = 1$ ko'rinishda bo'lishi kelib chiqadi.

Bu holning bo'lishi mumkin emas, chunki oxirgi tenglikning chap tomoni juft son va o'ng tomoni toq son. Bundan berilgan tenglamaning yechimi yo'qligi ma'lum bo'ladi.

Baholash usulidan foydalanib yechish.

Misol 3. $\dfrac{2tg\dfrac{x}{2}}{1+tg^2\dfrac{x}{2}}=k^2-4k+5$ tenglamani yeching.

Yechilishi:

$$\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}=k^2-4k+5 \quad \Leftrightarrow \quad \begin{cases} x\neq\pi+2\pi n,\ n\in Z \\ sin\,x=(k-2)^2+1 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x\neq\pi+2\pi n,\ n\in Z \\ sin\,x=1 \\ k=2 \end{cases} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \begin{cases} x\neq\pi+2\pi n,\ n\in Z \\ x=\frac{\pi}{2}+2\pi k,\ k\in Z \\ k=2 \end{cases} \Leftrightarrow x=\frac{\pi}{2}+2\pi k, k\in Z$$

$$Javob:\ x=\frac{\pi}{2}+2\pi k, k\in Z$$

TESKARI TRIGONOMETRIK FUNKSIYALAR QATNASHGAN TENGLAMALAR

Teskari trigonometrik funksiyalar qatnashgan eng sodda trigonometrik tenglamalar va ularning yechimlari quyidagi ko'rinishda bo'ladi:

$\arcsin x = a$, yechimi $x = \sin a$ ko'rinishda izlanadi va $-\frac{\pi}{2} \le a \le \frac{\pi}{2}$ shart o'rinli bo'lishi kerak. Xuddi shunday boshqa tenglamalarda ham:

$\arccos x = a$, $x = \cos a$, bu yerda $0 \le a \le \pi$,

$arctgx = a$, $x = tga$, bu yerda $-\frac{\pi}{2} < a < \frac{\pi}{2}$,

$arcctgx = a$, $x = ctga$, bu yerda $0 < a < \pi$ bo'ladi.

Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechish uchun quyidagi formulalarni bilib qo'ygan ma'qul.

$$\sin(\arcsin x) = x, \quad (1)$$

$$\cos(\arccos x) = x, \quad (2)$$

$$\sin(\arccos x) = \sqrt{1-x^2}, \quad (3)$$

$$\cos(\arcsin x) = \sqrt{1-x^2}, \quad (4)$$

$$\sin(arctgx) = \frac{x}{\sqrt{1+x^2}}, \quad (5)$$

$$\sin(arcctgx) = \frac{1}{\sqrt{1+x^2}}, \quad (6)$$

$$tg(\arcsin x) = \frac{x}{\sqrt{1-x^2}}, \quad (7)$$

$$tg(\arccos x) = \frac{\sqrt{1-x^2}}{x}, \quad (8)$$

$$tg(arctgx) = x, \quad (9)$$

$$\cos(arctgx) = \frac{1}{\sqrt{1+x^2}}, \quad (10)$$

$$\cos(arcctgx) = \frac{x}{\sqrt{1+x^2}}, \qquad (11)$$

$$ctg(\arcsin x) = \frac{\sqrt{1-x^2}}{x}, \qquad (12)$$

$$ctg(\arccos x) = \frac{x}{\sqrt{1-x^2}}, \qquad (13)$$

$$ctg(arctgx) = \frac{1}{x}, \qquad (14)$$

$$ctg(arcctgx) = x. \qquad (15)$$

Bulardan tashqari trigonometrik funksiyalar bilan bog'liq bo'lgan formulalar argumentda teskari trigonometrik funksiyalar bo'lgan hol uchun ham o'rinli. Jumladan,

$\sin 2x = 2\sin x \cdot \cos x$ formula teskari trigonometrik funksiyalar qatnashgan holda $\sin(2\arcsin x) = 2\sin(\arcsin x)\cos(\arcsin x)$ kabi,

$\cos 2x = \cos^2 x - \sin^2 x$ formula esa

$\cos(2\arccos x) = \cos^2(\arccos x) - \sin^2(\arccos x)$ kabi ifodalanadi.

Misol. $\sin(2\arccos\frac{3}{5})$ ning qiymatini toping.

Yechish: Ikkilangan burchakning sinusini topish formulasiga asosan:

$$\sin(2\arccos\frac{3}{5}) = 2\sin(\arccos\frac{3}{5})\cos(\arccos\frac{3}{5})$$

(2) va (3) formulalarga ko'ra

$$2\sin(\arccos\frac{3}{5}) \cdot \cos(\arccos\frac{3}{5}) = 2\sqrt{1-\frac{9}{25}} \cdot \frac{3}{5} = 2\sqrt{\frac{16}{25}} \cdot \frac{3}{5} = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}.$$

(Bu misolni $\arccos\frac{3}{5} = x, \quad \cos x = \frac{3}{5}$ ekanidan

$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{9}{25}}$ va $\sin 2x = 2\sin x \cos x$ lardan

foydalanib,

$\sin 2x = \sin(2\arccos\frac{3}{5}) = 2\sqrt{1-\frac{9}{25}} \cdot \frac{3}{5} = \frac{24}{25}$ deb ham yechish mumkin edi).

Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechish uchun agar tenglama tarkibida turli teskari trigonometrik funksiyalar qatnashgan bo'lsa yoki bu funksiyalar argumentlari har xil bo'lsa, berilgan tenglamani yechishda qulay holga keltirish uchun tenglikning ikkala tomonini bir xil trigonometrik funksiyalab olamiz.

Misol 1. $\arcsin x = \arccos x$ tenglamani yeching.

Yechish:

Tenglikning ikkala tomonining kosinusini olamiz:

$$\cos(\arcsin x) = \cos(\arccos x)$$

Yuqoridagi (2) va (4) formulaga asosan

$$\sqrt{1-x^2} = x.$$

Bu tenglamani yechib

$$x = \pm\frac{1}{\sqrt{2}}$$

bo'lishini topamiz.

Bu yerda $x = -\frac{1}{\sqrt{2}}$ begona ildizdir. Chunki oxirgi tenglamada tenglikning o'ng tomoni doimo musbat qiymatlarni qabul qiladi.

$x = \frac{1}{\sqrt{2}}$ ning ildiz bo'lishini tekshirib ko'rish kerak, ya'ni

$$\arcsin\frac{1}{\sqrt{2}} = \frac{\pi}{4},$$

$$\arccos\frac{1}{\sqrt{2}} = \frac{\pi}{4}.$$

Demak, $\frac{1}{\sqrt{2}}$ javob bo'la oladi.

Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechganda, ildizlarni albatta tekshirib ko'rish kerak (Ayniqsa, tangens va kotangensdan foydalanib hisoblaganda, bu funksiyalarning aniqlanish sohalariga kirmaydigan ildizlarning bor-yo'qligini tekshirib ko'rish kerak).

2. $\arccos x - \arcsin x = \frac{\pi}{6}$ tenglamani yeching.

Yechish: Tenglikning ikkala tomonidan kosinus olamiz.

$$\cos(\arccos x - \arcsin x) = \cos\frac{\pi}{6},$$

Tenglikning chap tomonini $\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$ formuladan foydalanib shakl almashtiramiz.

$$\cos(\arccos x)\cdot\cos(\arcsin x) + \sin(\arccos x)\cdot\sin(\arcsin x) = \frac{\sqrt{3}}{2}$$

Yuqorida keltirilgan formulalarga ko'ra

$$x\sqrt{1-x^2} + x\sqrt{1-x^2} = \frac{\sqrt{3}}{2},$$

$$2x\sqrt{1-x^2} = \frac{\sqrt{3}}{2}$$

ikkala tomonini kvadratga ko'tarib,

$$4x^2(1-x^2) = \frac{3}{4},$$

$$16x^2(1-x^2) = 3,$$

$$16x^4 - 16x^2 + 3 = 0$$

tenglamani hosil qilamiz. Bu tenglamaning ildizlari

$x_1 = \frac{1}{2}$, $x_2 = -\frac{1}{2}$, $x_3 = \frac{\sqrt{3}}{2}$, $x_4 = -\frac{\sqrt{3}}{2}$ lardan iborat bo'ladi.

Berilgan tenglamani yechishda birinchi galda tenglikning ikkala tomonidan kosinus olinib va ikkinchi galda irratsionallikdan qutulish uchun tenglikning ikkala tomonini kvadratga ko'tarish bilan ikki marta shakl almashtirish bajarildi. Bu shakl almashtirishlar esa chet ildizlarning paydo bo'lishiga olib kelishi mumkin. Shuning uchun topilgan ildizlarni tekshirib ko'ramiz. Tekshirishlar faqat $x_1 = \frac{1}{2}$ berilgan tenglamaning ildizi bo'lishini ko'rsatadi.

$-\frac{1}{2}$, $\frac{\sqrt{3}}{2}$ va $-\frac{\sqrt{3}}{2}$ lar chet ildizlardir.

3. $arctg\frac{1}{3} + arctg\frac{1}{5} + arctg\frac{1}{7} + arctg\frac{1}{8}$ ni hisoblang.

Yechilishi:

$\alpha_1 = arctg\frac{1}{3}$, $\alpha_2 = arctg\frac{1}{5}$, $\alpha_3 = arctg\frac{1}{7}$, $\alpha_4 = arctg\frac{1}{8}$ deb

belgilashlar kiritaylik. U holda $tg\alpha_1 = \frac{1}{3}$, $tg\alpha_2 = \frac{1}{5}$, $tg\alpha_3 = \frac{1}{7}$,

$tg\alpha_4 = \frac{1}{8}$ bo'ladi.

Berilgan yig'indini A deb belgilasak va ikkala tomonini tangenslasak, $tgA = tg(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)$ tenglik hosil bo'ladi.

$$tgA = \frac{tg(\alpha_1 + \alpha_2) + tg(\alpha_3 + \alpha_4)}{1 - tg(\alpha_1 + \alpha_2)tg(\alpha_3 + \alpha_4)}$$

$$tgA = \frac{\frac{tg\alpha_1 + tg\alpha_2}{1 - tg\alpha_1 tg\alpha_2} + \frac{tg\alpha_3 + tg\alpha_4}{1 - tg\alpha_3 tg\alpha_4}}{1 - \frac{tg\alpha_1 + tg\alpha_2}{1 - tg\alpha_1 tg\alpha_2} \cdot \frac{tg\alpha_3 + tg\alpha_4}{1 - tg\alpha_3 tg\alpha_4}} = \frac{\frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{3} \cdot \frac{1}{5}} + \frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \cdot \frac{1}{8}}}{1 - \frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{3} \cdot \frac{1}{5}} \cdot \frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \cdot \frac{1}{8}}} =$$

$$= \frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} \cdot \frac{3}{11}} = \frac{\frac{44 + 21}{77}}{\frac{77 - 12}{77}} = \frac{65}{65} = 1$$

Demak, $tgA = 1 \quad \Rightarrow \quad A = \frac{\pi}{4}$ Javob: $\frac{\pi}{4}$

Misollar yechish

1. (00-5-70). Ushbu $\sin(3x - 45^\circ) = \sin 14^\circ \cdot \sin 76^\circ - \cos 12^\circ \cdot \sin 16^\circ + \frac{1}{2}\cos 86^\circ$ tenglamaning $[0; 180^\circ]$ kesmadagi ildizlari yig'indisini toping.

A)135° B)150° C)210° D)215° E)225°

Yechilishi: Tenglamani yechish uchun

$$\sin x \cdot \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y)) \quad \text{va}$$

$$\sin x \cdot \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$

formulalardan foydalanamiz.

$$\sin 14^\circ \cdot \sin 76^\circ = \frac{1}{2}(\cos(14^\circ - 76^\circ) - \cos(14^\circ + 76^\circ)) = \frac{1}{2}(\cos(-62^\circ) - \cos 90^\circ) =$$

$$= \frac{1}{2}\cos 62^\circ = \frac{1}{2}\cos(90^\circ - 28^\circ) = \frac{1}{2}\sin 28^\circ \qquad (1)$$

$$\sin 16^\circ \cdot \cos 12^\circ = \frac{1}{2}(\sin(16^\circ + 12^\circ) + \sin(16^\circ - 12^\circ)) = \frac{1}{2}\sin 28^\circ + \frac{1}{2}\sin 4^\circ \qquad (2)$$

(1) va (2) larni dastlabki tenglamaga qo'yamiz:

$$\sin(3x - 45^\circ) = \frac{1}{2}\sin 28^\circ - \frac{1}{2}\sin 28^\circ - \frac{1}{2}\sin 4^\circ + \frac{1}{2}\cos 86^\circ;$$

$$\sin(3x - 45^\circ) = -\frac{1}{2}\sin 4^\circ + \frac{1}{2}\cos(90^\circ - 4^\circ);$$

$$\sin(3x - 45^\circ) = -\frac{1}{2}\sin 4^\circ + \frac{1}{2}\sin 4^\circ.$$

$$\sin(3x - 45^\circ) = 0, \qquad 3x - 45^\circ = k\pi, \qquad k \in Z.$$

$$3x = 45^\circ + 180^\circ, \qquad k \in Z, \qquad x = 15^\circ + 60^\circ, \qquad k \in Z.$$

Tenglamaning $[0; 180^\circ]$ kesmadagi ildizlari 15°, 75°, 135° dan iborat bo'lib, ularning yig'indisi 225° ga teng.

Javob:E

2. (00-9-36) Ushbu $a \cdot (\sin \quad x + \cos^6) = \sin^4 x + \cos^4 x$ tenglama ildizga ega bo'ladigan a ning barcha qiymatlarini toping.

A)[-1; 1] B)[0; 1] C)[1; 2] D)[1; 1,5] E)[1; 2,5]

Yechilishi: Ushbu tenglamani yechish uchun

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2), \quad \sin^2 x + \cos^2 x = 1, \quad \sin 2x = 2\sin x \cos x,$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

formulalardan hamda $y=\cos x$ funksiyaning xossasidan foydalanamiz. Tenglikning ikkala tomonini alohida-alohida soddalashtirib olamiz.

$$a((\sin^2 x)^3+(\cos^2 x)^3)=a(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x)=$$

$$=a(\sin^2 x+\cos^2 x)^2-3a\sin^2 x\cos^2 x=a-3a\sin^2 x\cdot\cos^2 x. \quad (1)$$

$$\sin^4 x+\cos^4 x=(\sin^2 x+\cos^2 x)-2\sin^2 x\cdot\cos^2 x=1-2\sin^2 x\cdot\cos^2 x=$$

$$=1-\frac{1}{2}(2\sin x\cdot\cos x)^2=1-\frac{\sin^2 2x}{2} \quad (2)$$

(1) va (2) larni berilgan tenglamaga qo'yamiz.

$$a-3a\sin^2 x\cos^2 x=1-\frac{\sin^2 2x}{2},$$

$$a-3\cdot\frac{a}{4}\cdot(4\sin^2 x\cdot\cos^2 x)=1-\frac{\sin^2 2x}{2},$$

$$a-\frac{3}{4}\cdot a\cdot\sin^2 2x=1-\frac{1}{2}\sin^2 2x. \qquad \sin^2 2x=\frac{1-\cos 4x}{2} \quad \textit{dan}$$

$$a-\frac{3}{4}\cdot a\cdot\frac{1-\cos 4x}{2}=1-\frac{1}{2}\cdot\frac{1-\cos 4x}{2},$$

$$8a-3a+3a\cos 4x=8-2+2\cos 4x$$

$$3a\cos 4x-2\cos 4x=6-5a$$

$$(3a-2)\cos 4x=6-5a, \quad \cos 4x=\frac{6-5a}{3a-2}.$$

Bu yerda $-1\le\frac{6-5a}{3a-2}\le 1$ bo'lishi kerak. Ushbu tengsizlikni yechib, javob [1; 2] to'plam ekanini topamiz.

Javob:C

3.(01-2-82) Ushbu $\sin x=\frac{2b-3}{4-b}$ tenglama b ning nechta butun qiymatida yechimga ega?

A)Ø B)1 C)2 D)3 E)4

Yechilishi: Tenglamaning o'ng tomonidagi ifoda $-1 \le \frac{2-3}{4-b} \le 1$ tengsizlikni qanoatlantirishi kerak. Ushbu tengsizlikning yichimi $b \in \left[-1; \frac{7}{3}\right]$ bo'lib, b ning masala shartini qanoatlantiruvchi butun qiymatlari -1,0,1,2 lardan iborat ekani kelib chiqadi.

Javob:E

4.(02-3-29) Agar $\sin\alpha$, $\sin 2\alpha$ va $\sin 3\alpha$ $(0 < \alpha < \pi)$ lar arifmetik progressiyani tashkil etsa, α ning qiymatini toping.

A) $\frac{\pi}{2}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{2\pi}{3}$

Yechilishi: $\sin\alpha$, $\sin 2\alpha$ va $\sin 3\alpha$ lar arifmetik progressiyani tashkil etgani uchun arifmetik progressiyaning ushbu $a_n = \frac{a_{n-1} + a_{n+1}}{2}$, ya'ni $2a_n = a_{n-1} + a_{n+1}$ xossasidan foydalanamiz.

$$\sin\alpha + \sin 3\alpha = 2\sin 2\alpha$$

Tenglikning chap tomoniga $\sin x + \sin y = 2\sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2}$ formulani qo'llab, $2\sin 2\alpha \cdot \cos\alpha = 2\sin 2\alpha$ tenglamani keltirib chiqaramiz va bundan α ni topamiz.

$$\sin 2\alpha(\cos\alpha - 1) = 0, \quad \sin 2\alpha = 0, \quad 2\alpha = k\pi, \quad \alpha = \frac{k\pi}{2}, \quad k \in Z.$$

$$\cos\alpha - 1 = 0, \quad \cos\alpha = 1, \quad \alpha = 2\pi n,\ n \in Z.$$

$0 < \alpha < \pi$ shartga asosan $\alpha = \frac{\pi}{2}$ ni olamiz.

Javob:A

5.(02-6-54). Uchburchakning α va β burchaklari orasida $\sin\alpha + \sin\beta = \sqrt{2}\cos\frac{\alpha-\beta}{2}$ munosabat o'rinli. Shu uchburchakning eng katta burchagini toping.

A) 120° B) 150° C) 90° D) 75° E) 100°

Yechilishi: $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ formuladan foydalanib, berilgan tenglikni $2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} = \sqrt{2}\cos\frac{\alpha-\beta}{2}$ ko'rinishga keltirib, yechamiz.

$$2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} - \sqrt{2}\cos\frac{\alpha-\beta}{2} = 0,$$

$$\cos\frac{\alpha-\beta}{2}(2\sin\frac{\alpha+\beta}{2} - \sqrt{2}) = 0,$$

$$2\sin\frac{\alpha+\beta}{2} = \sqrt{2}, \qquad \sin\frac{\alpha+\beta}{2} = \frac{\sqrt{2}}{2}, \qquad \frac{\alpha+\beta}{2} = \frac{\pi}{4}, \qquad \alpha+\beta = \frac{\pi}{2}.$$

$\alpha+\beta+\gamma=\pi$ bo'lgani uchun $\gamma = \frac{\pi}{2}$ – eng katta burchakdir.

Javob:C

6. (02-10-60). $\cos(\frac{3\pi+x}{3})\cdot\cos(\frac{9\pi+2x}{6}) = -\frac{5}{48}tg(2arctg1{,}5)$ tenglamani yeching.

A) $(-1)^{n+1}\frac{\pi}{4}+\frac{3\pi n}{2},\ n\in Z$ B) $(-1)^{n+1}\frac{\pi}{6}+\pi n,\ n\in Z$

C) $(-1)^{n}\frac{\pi}{3}+2\pi n,\ n\in Z$ D) $(-1)^{n}\frac{\pi}{6}+\frac{3\pi n}{2},\ n\in Z$

E) $\pm\frac{\pi}{3}+2\pi n,\ n\in Z.$

Yechilishi: Keltirish formulalaridan foydalanib, $\cos(\frac{3\pi+x}{3})$ va $\cos(\frac{9\pi+2x}{6})$ larni soddalashtirib olamiz.

$$\cos(\frac{3\pi+x}{3}) = \cos(\pi+\frac{x}{3}) = -\cos\frac{x}{3},$$

$$\cos(\frac{9\pi+2x}{6}) = \cos(\frac{3\pi}{2}+\frac{x}{3}) = \sin\frac{x}{3}.$$

Endi $tg(2arctg1{,}5)$ ifodani $tg(2arctgx)=\frac{2x}{1-x^2}$ formulaga ko'ra $tg(2arctg1{,}5)=-\frac{12}{5}$ ekanidan topilganlarni berilgan tenglamaga qo'yamiz.

$$-\cos\frac{x}{3}\cdot\sin\frac{x}{3}=-\frac{5}{48}\cdot(-\frac{12}{5}).$$

$\sin 2x=2\sin x\cdot\cos x$ formuladan $\cos\frac{x}{3}\cdot\sin\frac{x}{3}=\frac{1}{2}\sin\frac{2x}{3}$. U holda

$\frac{1}{2}\sin\frac{2x}{3}=-\frac{1}{4}$, $\quad\quad\quad \sin\frac{2x}{3}=-\frac{1}{2}$;

$\frac{2x}{3}=(-1)^{n+1}\frac{\pi}{6}+\pi n$, $\quad\quad\quad x=(-1)^{n+1}\frac{\pi}{4}+\frac{3\pi n}{2}$, $n\in Z$.

Javob:A

7.(02-11-43). $3\sin^2 2x+7\cos 2x-3=0$ tenglamaning $(-90^\circ;\ 180^\circ)$ intervalga tegishli ildizlari yig'indisini toping.

A)90° B)105° C)180° D)135° E)150°

Yechilishi: $\sin^2 2x=1-\cos^2 2x$ deb o'rniga qo'yib, yechishni davom qildiramiz.

$$3(1-\cos^2 2x)+7\cos 2x-3=0,$$
$$-3\cos^2 2x+7\cos 2x=0,$$
$$\cos 2x(3\cos 2x-7)=0,$$
$$\cos 2x=0,\quad 2x=\frac{\pi}{2}+\pi n,\quad n\in Z,$$

$x=\frac{\pi}{4}+\frac{n\pi}{2}$, $n\in Z$. Bu yechimlardan $(-90^\circ;\ 180^\circ)$ oraliqqa tegishlilari $-\frac{\pi}{4}$, $\frac{\pi}{4}$, $\frac{3\pi}{4}$ lardan iborat, ularning yig'indisi

$$-\frac{\pi}{4}+\frac{\pi}{4}+\frac{3\pi}{4}=\frac{3\pi}{4},\text{ ya'ni } 135^\circ.$$

($\cos 2x \neq \frac{7}{3}$ bo'lgani uchun qaralmadi.)

Javob:D

8.(03-1-24). Agar $|\cos x| = 2 + \cos x$ bo'lsa, $2^{\cos x} + 2^{\sin x}$ ning qiymatini toping.

A)1 B)0,5 C)0,7 D)1,25 E)1,5

Yechilishi: Tenglamada ikkita holni qaraymiz:

a) $\cos x \geq 0$ bo'lsin. U holda $\cos x = 2 + \cos x$ bo'lib, bu holning bo'lishi mumkin emas.

b) $\cos x < 0$ bo'lsa,

$-\cos x = 2 + \cos x$, $\quad -2\cos x = 2$, $\quad \cos x = -1$.

$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-1} = 0$,

u holda $2^{\cos x} + 2^{\sin x} = 2^{-1} + 2^{0} = \frac{1}{2} + 1 = \frac{3}{2} = 1{,}5$.

Javob:E

9. (03-2-30). $3^{\cos x} \cdot 3^{\cos^2 x} \cdot 3^{\cos^3 x} \cdot ... = 3$ tenglamani yeching.

A) $\pm\frac{\pi}{3} + 2\pi k,\ k \in Z$ B) $\frac{\pi}{3} + \pi k,\ k \in Z$

C) $\frac{2\pi}{3} + \pi k,\ k \in Z$ D) $\pm\frac{\pi}{6} + 2\pi k,\ k \in Z$

E) $(-1)^k \frac{\pi}{3} + \pi k,\ k \in Z$

Yechilishi:

$$3^{\cos x} \cdot 3^{\cos^2 x} \cdot 3^{\cos^3 x} \cdot ... = 3,$$

$$3^{\cos x + \cos^2 x + \cos^3 x + ...} = 3^1,$$

$$\cos x + \cos^2 x + \cos^3 x + ... = 1.$$

Tenglikning chap tomoni $b_1 = \cos x$, $q = \cos x$ bo'lgan geometrik progressiyaning hadlari yig'indisidan iborat. $|\cos x| \leq 1$ bo'lgani uchun bu yig'indini hisoblashda cheksiz kamayuvchi geometrik progressiya hadlari yig'indisini topish formulasi $S = \frac{b_1}{1-q}$ dan foydalanamiz.

$$S = \frac{\cos x}{1-\cos x}, \quad \frac{\cos x}{1-\cos x} = 1, \quad 2\cos x = 1, \quad \cos x = \frac{1}{2}, \quad x = \pm\frac{\pi}{3} + 2\pi k, \quad k \in Z.$$

Javob:A

10.(03-5-42). $\cos^2(\frac{\pi x}{6}) + \sqrt{2x^2 - 5x - 3} = 0$ tenglamani yeching.

A)3 B) $\frac{3}{2}$ C) $-\frac{1}{2}$ D)-3 E) $\frac{1}{2}$.

Yechilishi:

Qo'shiluvchilarning ikkalasi ham nomanfiy bo'lgani uchun

$$\begin{cases} \cos^2(\frac{\pi x}{6}) = 0, \\ 2x^2 - 5x - 3 = 0, \end{cases}$$ tenglamalar sistemasini yechishga keltiriladi.

$x = 3 + 6n$, $n \in Z$

$$\begin{cases} 1 + \cos\frac{\pi x}{3} = 0, \\ 2(x + \frac{1}{2})(x - 3) = 0. \end{cases} \Rightarrow \begin{cases} \cos\frac{\pi x}{3} = -1, \\ x_1 = -\frac{1}{2}, \\ x_2 = 3, \end{cases} \Rightarrow \begin{cases} \frac{\pi x}{3} = \pi, \\ x_1 = -\frac{1}{2}, \\ x_2 = 3, \end{cases} \Rightarrow \begin{cases} x_3 = 3, \\ x_1 = -\frac{1}{2}, \\ x_2 = 3. \end{cases}$$

$x_1 = -\frac{1}{2}$ da $\cos^2(-\frac{\pi}{12}) \neq 0$, demak tenglamani qaonoatlantirmaydi. Tenglamani x 3 javob qanoatlantiradi.

Javob:A

11. (03-6-63). Qanday eng kichik o'tkir burchak $\sin(2x+45^\circ)=\cos(30^\circ-x)$ tenglamani qanoatlantiradi?

A)25° B)5° C)45° D)30° E)15°

Yechilishi:

$\sin(2x+45^\circ)-\cos(30^\circ-x)=0$,

$\sin(2x+45^\circ)-\sin(90^\circ-(30^\circ-x))=0$,

$\sin(2x+45^\circ)-\sin(60^\circ+x)=0$,

$\sin(2x+45^\circ)-\sin(60^\circ+x)=2\sin\frac{2x+45^\circ-60^\circ-x}{2}\cdot\cos\frac{2x+45^\circ+60^\circ+x}{2}=$

$=2\sin\frac{x-15^\circ}{2}\cdot\cos\frac{3x+105^\circ}{2}$ bo'lgani uchun

$2\sin\frac{x-15^\circ}{2}\cdot\cos\frac{3x+105^\circ}{2}=0$, $\sin\frac{x-15^\circ}{2}=0\Rightarrow x=360^\circ+15^\circ$, $n\in Z$ $x=15^\circ$.

$\cos\frac{3x+105}{2}=0\Rightarrow x=120^\circ+25^\circ$, $n\in Z$ $x=25$.

Javob:E

12. (03-12-61). a parametrning qanday qiymatlarida $\sin^6 x+\cos^6 x=a$ tenglama yechimga ega?

A)[0; 1] B)[0,5; 1] C)[0,25; 0.5] D)[0,25; 1] E)[0,25; 0,75].

Yechilishi:

Tenglamani yechish uchun $a^3+b^3=(a+b)(a^2-ab+b^2)$ formuladan foydalanamiz:

$(\sin^2 x)^3+(\cos^2 x)^3=a$,

$(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x)=a$,

$(\sin^4 x+2\sin^2 x\cdot\cos^2 x+\cos^4 x)-3\sin^2 x\cdot\cos^2 x=a$,

$(\sin^2 x+\cos^2 x)^2-3\sin^2 x\cdot\cos^2 x=a$,

$1-\frac{3}{4}\sin^2 2x=a$, $\sin^2 2x=\frac{1-\cos 4x}{2}$ formulaga ko'ra

$1-\frac{3}{8}(1-\cos 4x)=a$,

$3\cos 4x=8a-5$, $\cos 4x=\frac{8}{3}a-\frac{5}{3}$,

$-1\le\cos x\le 1$ bo'lgani uchun

$-1\le\frac{8}{3}a-\frac{5}{3}\le 1$, $2\le 8a\le 8$, $\frac{1}{4}\le a\le 1$, ya'ni [0,25; 1] to'plam yechim bo'ladi.

Javob:D

13. (03-102-36). $\frac{3}{5}\log_{\sin^3 x}\cos^5 2x-3+2\log_{\cos 2x}\sin x=0$ tenglamani yeching.

A) $(-1)^k\frac{\pi}{6}+\pi k;\ (-1)^k\arcsin\frac{1}{\sqrt{3}}+\pi k,\ k\in Z$

B) $\pm\arcsin\frac{1}{\sqrt{3}}+\pi k,\ k\in Z$

C) $\pm\frac{\pi}{6}+\pi k;\pm\arcsin\frac{1}{\sqrt{3}}+\pi k,\ k\in Z$

D) $\pm\frac{\pi}{6}+\pi k,\ k\in Z$

E) $\frac{\pi}{6}+2\pi k;\ \arcsin\frac{1}{\sqrt{3}}+2\pi k,\ k\in Z$

Yechilishi: $\frac{3}{5}\cdot\frac{1}{3}\cdot 5\log_{\sin x}\cos 2x-3+2\log_{\cos 2x}\sin x=0$

$\log_{\sin x}\cos 2x=a$ belgilashni kiritamiz, bunda

$\sin x>0,\ \cos 2x>0,\ \sin x\ne 1,\ \cos 2x\ne 1$

$$a^2-3a+2=0 \quad \Rightarrow \quad a_1=1,\ a_2=2$$

1) $$\log_{\sin x}\cos 2x=1$$

$$\cos 2x=\sin x$$

$$2\sin^2 x+\sin x-1=0$$

a) $$\sin x=\frac{1}{2}$$

$$x=(-1)^n\arcsin\frac{1}{2}+\pi k$$

$$x=(-1)^n\frac{\pi}{6}+\pi k,\ k\in Z$$

b) $$\sin x=-1$$

$x\in\varnothing$, *chunki*

$\sin x>0$

shartga zid

2) $$\log_{\sin x}\cos 2x=2$$

$$\cos 2x=\sin^2 x$$

$$\sin^2 x=\frac{1}{3}$$

a) $$\sin x=\frac{1}{\sqrt{3}}$$

$$x=(-1)^k\arcsin\frac{1}{\sqrt{3}}+\pi k,\ k\in Z$$

b) $$\sin x=-\frac{1}{\sqrt{3}}$$

$x\in\varnothing$, *chunki*

$\sin x>0$

shartga zid

Aniqlanish sohasiga e'tibor bersak, $\frac{\pi}{6}+2\pi k;\ \arcsin\frac{1}{\sqrt{3}}+2\pi k,\ k\in Z$ javobni hosil qilamiz.

Javob: E.

14. (03-108-36). $16\sin^2 x\cos^2 x-\sin^6 x=\cos^6 x$ tenglamani yeching.

A) $\pm\frac{1}{2}\arcsin\frac{2}{\sqrt{19}}+\frac{k\pi}{2},\ k\in Z$

B) $\pm\arcsin\frac{3}{\sqrt{19}}+2k\pi,\ k\in Z$

C) $\pm\arcsin\frac{2}{\sqrt{17}}+k\pi,\ k\in Z$

D) $\pm\arcsin\frac{\sqrt{2}}{\sqrt{19}}+k\pi,\ k\in Z$

Yechilishi:

$$16\sin^2 x\cos^2 x-(\sin^6 x+\cos^6 x)=0\ \Rightarrow\ 4\sin^2 2x-1+\frac{3}{4}\sin^2 2x=0\ \Rightarrow$$

$$\Rightarrow\ \frac{19}{4}\sin^2 2x=1\ \Rightarrow\ \sin^2 2x=\frac{4}{19}\Rightarrow\ 2x=\pm\arcsin\frac{2}{\sqrt{19}}+k\pi\ \Rightarrow$$

$$\Rightarrow\ x=\pm\frac{1}{2}\arcsin\frac{2}{\sqrt{19}}+\frac{k\pi}{2},\ k\in Z$$

Yuqorida $\sin^6 x+\cos^6 x=1-\frac{3}{4}\sin^2 2x$ ekanidan foydalandik.

Javob: A.

15. (03-117-36). $1-\cos^6 x=\sin^5 x$ tenglamaning $\left[-\frac{7\pi}{4};\frac{5\pi}{4}\right]$ kesmadagi eng katta va eng kichik ildizlari orasidagi ayirmani toping.

A) 3π B) $3{,}5\pi$ C) $1{,}5\pi$ D) 2π E) $2{,}5\pi$

Yechilishi:

Tenglamani $\cos^6 x+\sin^5 x=1$ ko'rinishda yozib olib, $|\cos x|\le 1, |\sin x|\le 1$ ekanligidan mantiqan fikrlab, $\cos x=\pm 1;\ \sin x=1$ tenglamalarga egamiz, ularni esa alohida yechib olamiz.

$\cos x = \pm 1$

$x = 2\pi n$

n	0
x	0

$x = \pi + 2\pi n, n \in Z$

n	0	-1
x	π	$-\pi$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n, n \in Z$

n	0	-1
x	$\frac{\pi}{2}$	$-\frac{3\pi}{2}$

Endi masala shartidan $\pi - \left(-\frac{3\pi}{2}\right) = \pi + \frac{3\pi}{2} = \pi + 1{,}5\pi = 2{,}5\pi$

Javob: E.

16. (03-118-36).

$\sin(4x - 60^0) = \sin 14^0 \sin 76^0 - \cos 12^0 \sin 16^0 + \frac{1}{2}\cos 86^0$ tenglamaning $[0^0; 180^0]$ kesmadagi ildizlari yig'indisini toping.

A) 215^0 B) 210^0 C) 330^0 D) 135^0 E) 225^0

Yechilishi: Oldin tenglikni o'ng tomonida soddalashtirishni amalga oshiramiz. Buning uchun $\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$, $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$

$\cos\alpha = \sin(90^0 - \alpha)$ formulalardan foydalanamiz.

$\sin(4x - 60^0) = \frac{1}{2}(\cos 62^0 - \cos 90^0 - \sin 28^0 - \sin 4^0 + \sin 4^0)$.

$(\cos 62^0 = \cos(90^0 - 28^0) = \sin 28^0)$ $\quad \sin(4x - 60^0) = 0 \Rightarrow$

$\Rightarrow \quad 4x - 60^0 = 180^0 n \quad \Rightarrow \quad 4x = 60^0 + 180^0 n \quad \Rightarrow \quad x = 15^0 + 45^0 n,\ n \in Z$

n	0	1	2	3
x	15^0	60^0	105^0	150^0

Endi masala shartiga ko'ra
$15^0 + 60^0 + 105^0 + 150^0 = 330^0$

Javob: C.

17. (03-121-32). $tgx - tg\frac{\pi}{3} - tgxtg\frac{\pi}{3} = 1$ tenglamani yeching

A) $\frac{7\pi}{12} + 2\pi k,\ k \in Z$ B) $\frac{5\pi}{6} + \pi k,\ k \in Z$

C) $\frac{7\pi}{6} + \pi k,\ k \in Z$ D) $\frac{7\pi}{12} + \pi k,\ k \in Z$

E) $\frac{5\pi}{6} + 2\pi k,\ k \in Z$

Yechilishi:

$$tgx - tg\frac{\pi}{3} = 1 + tgxtg\frac{\pi}{3} \quad \Rightarrow \quad \frac{tgx - tg\frac{\pi}{3}}{1 + tgxtg\frac{\pi}{3}} = 1 \quad \Rightarrow \quad tg\left(x - \frac{\pi}{3}\right) = 1 \quad \Rightarrow$$

$$\Rightarrow \quad x = \frac{7\pi}{12} + \pi k,\ k \in Z$$

Javob: D.

18. (04-105-32). $4\sin^2 x(1 + \cos 2x) = 1 - \cos 2x$ tenglamani yeching

A) $\pm\frac{\pi}{3} + \pi n,\ n \in Z$ B) $\pi n,\ \pm\frac{2\pi}{3} + 2\pi n, n \in Z$

C) $\pi n, n \in Z$ D) $\pi n,\ \pm\frac{\pi}{3} + 2\pi n, n \in Z$

E) $\pi n,\ \pm\frac{\pi}{3} + \pi n, n \in Z$

Yechilishi:

$4\sin^2 x\cdot 2\cos^2 x = 1-\cos 2x \quad\Rightarrow\quad 2\sin^2 2x = 1-\cos 2x \quad\Rightarrow$

$\Rightarrow\; 2(1-\cos^2 2x) = 1-\cos 2x \;\Rightarrow\; 2\cos^2 2x-\cos 2x-1=0,\; \cos 2x = a$

belgilash bilan $2a^2-a-1=0$ tenglamaga keltirib yechiladi. $a_1=1,\ a_2=-\frac{1}{2}$.

Endi belgilashga qaytib quyidagilarga ega bo'lamiz:

1) $\cos 2x = 1$

$2x = 2\pi n$

$x=\pi n,\ n\in Z$

2) $\cos 2x = -\frac{1}{2}$

$2x=\pm\frac{2\pi}{3}+2\pi n$

$x=\pm\frac{\pi}{3}+\pi n,\ n\in Z$

Javob: E.

19. (04-107-32). $|tgx+ctgx|=\frac{4}{\sqrt{3}}$ tenglamani yeching

A) $(-1)^n\frac{\pi}{6}+2\pi k,\ k\in Z$

B) $\frac{2\pi}{3}+\pi k,\ k\in Z$

C) $\pm\frac{\pi}{6}+\frac{\pi k}{2},\ k\in Z$

D) $\pm\frac{\pi}{3}+\pi k,\ k\in Z$

E) $\frac{\pi}{3}+2\pi k,\ k\in Z$

Yechilishi:

$\left|\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right|=\frac{4}{\sqrt{3}} \Rightarrow \frac{2}{|\sin 2x|}=\frac{4}{\sqrt{3}} \Rightarrow |\sin 2x|=\frac{\sqrt{3}}{2} \Rightarrow 2x=\pm\frac{\pi}{3}+\pi k \Rightarrow$

$\Rightarrow x=\pm\frac{\pi}{6}+\frac{\pi k}{2},\ k\in Z$

Javob: C.

20. (04-111-32). $\sqrt{\cos x}\cdot\sin x=0$ tenglamani yeching. $\cos x \geq 0$.

A) $2\pi k,\ \frac{\pi}{2}+\pi k,\ k\in Z$ B) $\pi+2\pi k,\ k\in Z$

C) $\frac{\pi}{2}+\pi k,\ k\in Z$ D) $\frac{\pi}{2}+2\pi k,\ k\in Z$

E) $\pi k,\ k\in Z$

Yechilishi: Ko'paytmaning nolga tengligi sharti va $\cos x>0$ ekanligiga e'tibor berib, ushbu sistemaga egamiz:

$$\begin{cases}\cos x=0\\ \sin x=0\\ \cos x>0\end{cases} \Rightarrow \begin{cases}x=\frac{\pi}{2}+\pi k,\ k\in Z\\ x=\pi k,\ k\in Z\\ \cos x>0\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}x=\frac{\pi}{2}+\pi k,\ k\in Z\\ x=2\pi k,\ k\in Z\end{cases}$$

Javob: A.

21. (04-119-32). $\sin x=\frac{2b-3}{4-b}$ tenglamani b ning nechta butun qiymatida yechimga ega bo'ladi?

A) 2 B) 4 C) Ø D) 3 E) 1

Yechilishi: Ma'lumki $|\sin x|\leq 1$. U holda $\left|\frac{2b-3}{4-b}\right|\leq 1$ bo'lishi shart.

$$\begin{cases}\frac{2b-3}{4-b}\leq 1\\ \frac{2b-3}{4-b}\geq -1\end{cases} \Leftrightarrow \begin{cases}b\leq\frac{7}{3},\ b\geq 4\\ -1\leq b\leq 4\end{cases} \Leftrightarrow b\in\left[-1;\frac{7}{3}\right]$$

Bu oraliqda 4 ta butun yechim bor.

Javob: B.

22. (05-103-24). $\dfrac{\cos 2x}{\dfrac{\sqrt{2}}{2}+\sin x}=0$ tenglamaning $[0, 6\pi]$ kesmada necha ildizi bor?

A) 4 B) 12 C) 8 D) 2 E) 6

Yechilishi:

$$\begin{cases} \cos 2x = 0 \\ \dfrac{\sqrt{2}}{2}+\sin x \neq 0 \end{cases} \Leftrightarrow \begin{cases} \cos 2x = 0 \\ \sin x \neq -\dfrac{\sqrt{2}}{2} \end{cases} \Rightarrow$$

$$\cos 2x = 0$$

$$x=\frac{\pi}{4}+\frac{\pi n}{2},\ n\in Z$$

n	0	1	2	3	4	5	6	7	8	9	10	11
x	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$	$\frac{11\pi}{4}$	$\frac{13\pi}{4}$	$\frac{15\pi}{4}$	$\frac{17\pi}{4}$	$\frac{19\pi}{4}$	$\frac{21\pi}{4}$	$\frac{23\pi}{4}$

$\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4}, \frac{21\pi}{4}$ lar chet ildiz. Qolgan ildizlar soni 6 ta.

Javob: E.

23. (05-114-24). $\dfrac{1+\cos x}{\sin x}=2\cos\dfrac{x}{2}$ tenglamaning $\left[0;\dfrac{16\pi}{3}\right]$ kesmada nechta ildizi bor?

A) 2 B) 4 C) Ø D) 3 E) 1

Yechilishi: $1+\cos x = 2\cos^2\frac{x}{2}$ tenglikdan foydalanamiz, $\sin x \neq 0$ ekanligini yodda tutgan holda

$$\frac{2\cos^2\frac{x}{2}}{\sin x} = 2\cos\frac{x}{2} \Rightarrow \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} - 2\cos\frac{x}{2} = 0 \Rightarrow$$

$$\Rightarrow \cos\frac{x}{2} - 2\cos\frac{x}{2}\sin\frac{x}{2} = 0 \Rightarrow \cos\frac{x}{2}\left(1-2\sin\frac{x}{2}\right) = 0$$

1) $\frac{x}{2} = \frac{\pi}{2} + \pi k$

$x = \pi + 2\pi k,\ k \in Z$

2) $\sin\frac{x}{2} = \frac{1}{2}$

$x = (-1)^n\frac{\pi}{3} + 2\pi k,\ k \in Z$

Bunda chet ildizlar hosil bo'ladi

n	0	1	2
x	$\frac{\pi}{3}$	$\frac{5\pi}{3}$	$\frac{13\pi}{3}$

Javob: D.

24. (05-116-24). $\frac{\sin 2x}{ctgx - \cos x} = 0$ tenglamani yeching.

A) $2\pi k,\ k \in Z$ B) $\pi k,\ k \in Z$ C) $\frac{\pi k}{2},\ k \in Z$ D) Ø

E) $\frac{\pi}{2} + \pi k,\ k \in Z$

Yechilishi:

$$\begin{cases} \sin 2x = 0 \\ \sin x \neq 0 \\ \cos x \neq 0 \end{cases} \Leftrightarrow \begin{cases} 2\sin x\cos x = 0 \\ \sin x \neq 0 \\ \cos x \neq 0 \end{cases} \Rightarrow x \in \varnothing$$

Javob: D.

25. **(05-123-24).** $4\sin\frac{x}{2}-\cos x+1=0$ tenglamaning $[0,8\pi]$ kesmada necha ildizi bor?

A) 3 B) 4 C) 5 D) 1 E) 2

Yechilishi:

$$4\sin\frac{x}{2}-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+\sin^2\frac{x}{2}=0 \Rightarrow 2\sin^2\frac{x}{2}+4\sin\frac{x}{2}=0 \ |:2 \Rightarrow$$

$$\Rightarrow \sin\frac{x}{2}\left(\sin\frac{x}{2}+2\right)=0$$

1) $\sin\frac{x}{2}=0$

$\frac{x}{2}=\pi n$

$x=2\pi n,\ n\in Z$

2) $\sin\frac{x}{2}+2=0$

$\sin\frac{x}{2}\neq -2$

$x\in\varnothing$

n	0	1	2	3	4
x	0	2π	4π	6π	8π

Javob: C.

26. (05-124-32). $\cos x\cdot\cos 4x-\cos 5x=0$ tenglama $[0;\ \pi]$ kesmada nechta ildizga ega?

A)4 B)5 C)1 D)3 E)2

Yechilishi: Ushbu $\cos x\cdot\cos y=\frac{1}{2}(\cos(x+y)+\cos(x-y))$ formuladan foydalanib, berilgan tenglamani quyidagi ko'rinishga keltirib yechishni davom ettiramiz:

$$\frac{1}{2}\cos 5x+\frac{1}{2}\cos 3x-\cos 5x=0,$$

$$\frac{1}{2}\cos 3x-\frac{1}{2}\cos 5x=0, \qquad \frac{1}{2}(\cos 3x-\cos 5x)=0,$$

$\cos 3x - \cos 5x = 0$. Endi $\cos x - \cos y = -2\sin\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$ formulaga asosan oxirgi tenglamani ko'paytmaga keltirib olamiz

$-2\sin 4x \cdot \sin(-x) = 0$, $\sin x \cdot \sin 4x = 0$,

$$\sin x = 0,\ x = \pi n,\ n \in Z.$$

$$\sin 4x = 0,\ 4x = k\pi,\ x = \frac{k\pi}{4},\ k \in Z.$$

Bulardan $[0; \pi]$ oraliqda berilgan tenglamaning 5 ta ildizi borligi kelib chiqadi, ya'ni k=0, 1, 2, 3, 4 bo'lgan hollarda.

Javob:B.

27. (05-135-32). $\frac{\cos^2 x - \cos x}{\sin x} = 0$ tenglama $[-2\pi;2\pi]$ oraliqda necha ildizga ega?

A) 3 B) 1 C) 6 D) 2 E) 4

Yechilishi:

$$\frac{a}{b} = 0 \Leftrightarrow \begin{cases} a = 0 \\ b \neq 0 \end{cases}$$ munosabatdan foydalanamiz.

$$\begin{cases} \cos^2 x - \cos x = 0 \\ \sin x \neq 0 \end{cases} \Rightarrow \cos x(\cos x - 1) = 0$$

1) $\cos x = 0$

$$x = \frac{\pi}{2} + \pi k,\ k \in Z$$

2) $\cos x - 1 = 0$

$\cos x = 1$

Bu holni qaramaymiz, chunki *sinx=0*

k	0	1	-1	-2
x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{3\pi}{2}$

Javob: E.

28. (05-137-24). $(1+\cos x)tg\frac{x}{2}+1=0$ tenglamani yeching.

A) $-\frac{\pi}{2}+2\pi k, k\in Z$ B) $\pi+2\pi k, k\in Z$

C) $\pi k, k\in Z$ D) $\pi+\pi k, k\in Z$

E) $\frac{\pi}{2}+2\pi k, k\in Z$

Yechilishi:

$$\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)tg\frac{x}{2}+1=0 \Rightarrow 2\cos^2\frac{x}{2}\cdot\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}+1=0 \Rightarrow$$

$$\Rightarrow \sin x+1=0 \Rightarrow \sin x=-1 \Rightarrow x=-\frac{\pi}{2}+2\pi k,\ k\in Z$$

Javob: A.

29. (05-141-24). Agar $\sin\alpha\cdot\cos\beta=1-0{,}5\sqrt{3}$ va $\sin\beta\cdot\cos\alpha=1$ bo'lsa, $\alpha-\beta$ ning qiymatini toping.

A) $(-1)^k\frac{\pi}{3}+2\pi k,\ k\in Z$ B) $(-1)^k\frac{\pi}{3}+\pi k,\ k\in Z$

C) $(-1)^{k+1}\frac{\pi}{3}+\pi k,\ k\in Z$ D) $(-1)^k\frac{\pi}{6}+\pi k,\ k\in Z$

Yechilishi:

Birinchi va ikkinchi tengliklarni hadlab ayiramiz:
$\sin\alpha\cdot\cos\beta-\sin\beta\cdot\cos\alpha=-0{,}5\cdot\sqrt{3}$.

Tenglikning chap tomoni ikki burchak ayirmasining sinusi formulasiga tushadi. U holda $\sin(\alpha-\beta)=-\frac{\sqrt{3}}{2}$.

$\alpha-\beta=(-1)^{k+1}\frac{\pi}{3}+\pi k,\ k\in Z$.

Javob:C.

30. (06-102-33). $\sin 5x - 3\cos 2x = 4$ tenglamani yeching.

A) $\frac{\pi}{2} + \pi n,\ n \in Z$ B) $-\frac{\pi}{2} + 2\pi n,\ n \in Z$

C) $\frac{\pi}{2} + 2\pi n,\ n \in Z$ D) $\pi + \pi n,\ n \in Z$

Yechilishi: Bu tenglamani nostandart usuldan foydalanib yechish qulay va vaqtni tejaydi. Bu yerda $\sin x$ va $\cos x$ larning chegaralanganligidan foydalanamiz, ya'ni $-1 \leq \sin 5x \leq 1$ va $-1 \leq \cos 2x \leq 1$ bo'lganda tenglikning chap qismi 3dan ortib ketolmaydi, 3ga teng bo'ladi, agar $\begin{cases} \sin 5x = 1 \\ \cos 2x = -1 \end{cases}$ bo'lsa, x ning ikkala tenglamani qanoatlantiruvchi qiymatlarini topish uchun ulardan bittasini yechib, bu yechimdan ikkinchi tenglamani qanoatlantiruvchi x ning qiymatlarini ajratib olamiz. Ikkinchi $\cos 2x = -1$ tenglamani yechaylik, $2x = \pi + 2k\pi$, $x = \frac{\pi}{2} + k\pi$, $k \in Z$.

Bundan $5x = \frac{5\pi}{2} + 5\pi k$, $\sin 5x = \sin(\frac{5\pi}{2} + 5\pi k) = \sin(\frac{\pi}{2} + \pi k)$.

Oxirgi tenglikdan ko'rinib turibdiki, k faqat juft qiymatlarni qabul qilganda $\sin 5x = 1$ tenglik o'rinli bo'lar ekan. U holda javob $x = \frac{\pi}{2} + 2\pi n$, $n \in Z$ bo'ladi.

Javob:C.

31. (06-119-33). $\cos 2x - 5\sin x - 3 = 0$ tenglamani yeching.

A) $(-1)^{n+1}\frac{\pi}{6} + \pi n,\ n \in Z$ B) $(-1)^{n}\frac{\pi}{6} + \pi n,\ n \in Z$

C) $(-1)^{n+1}\frac{\pi}{6} + 2\pi n,\ n \in Z$ D) $(-1)^{n}\frac{\pi}{6} + 2\pi n,\ n \in Z$

Yechilishi:

$\cos 2x = \cos^2 x - \sin^2 x$ va $\cos^2 x = 1 - \sin^2 x$ formulalardan foydalanib, tenglamani faqat $\sin x$ orqali ifodalab olamiz.

$2\sin^2 x+5\sin x+2=0$, $\sin x=t$ deb belgilab. $2t^2+5t+2=0$ kvadrat tenglamani yechamiz. Bu yerda $t_1=-2$ va $t_2=-\frac{1}{2}$ bo'lib, $\sin x=-2$ ma'noga ega emas, demak $\sin x=-\frac{1}{2}$ dan $x=(-1)^{n+1}\frac{\pi}{6}+\pi n$, $n\in Z$.

Javob:A.

32. (06-142-33). $|tgx+ctgx|=\frac{4}{\sqrt{3}}$ tenglamani yeching.

A) $\frac{\pi}{3}+2\pi k,\ k\in Z$ B) $\pm\frac{\pi}{6}+\frac{\pi k}{2},\ k\in Z$

C) $\pm\frac{\pi}{3}+\pi k,\ k\in Z$ D) $(-1)^k\frac{\pi}{6}+2\pi k,\ k\in Z$

Yechilishi:

$$\left|\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right|=\frac{4}{\sqrt{3}},\qquad \left|\frac{2}{\sin 2x}\right|=\frac{4}{\sqrt{3}}.$$

Bu yerda ikki holni qaraymiz.

a) $\sin 2x>0$ bo'lsa,

$$\frac{2}{\sin 2x}=\frac{4}{\sqrt{3}},\ \sin 2x=\frac{\sqrt{3}}{2},\ 2x=\frac{\pi}{3}+k\pi,\ k\in Z,\ x=\frac{\pi}{6}+\frac{k\pi}{2},\ k\in Z.$$

b) Agar $\sin 2x<0$ bo'lsa, $-\frac{2}{\sin 2x}=\frac{4}{\sqrt{3}}$, $\sin 2x=-\frac{\sqrt{3}}{2}$ bundan

$$2x=-\frac{\pi}{3}+k\pi,\quad x=-\frac{\pi}{6}+\frac{k\pi}{2},\quad k\in Z.$$

Demak, bularni umumlashtirib, $x=\pm\frac{\pi}{6}+\frac{\pi k}{2}$, $k\in Z$ javobni olamiz.

Javob:B.

33. (07-113-24). $4\cos 5x = 6 + 3\cos\left(\frac{\pi}{2} + 5x\right)$ tenglama $[-\pi; 2\pi]$ kesmada nechta yechimga ega?

A) 1 B) Ø C) 3 D) 2

Yechilishi:

$4\cos 5x = 6 - 3\sin 5x \quad \Rightarrow \quad 4\cos 5x + 3\sin 5x = 6 \quad \Rightarrow \quad \frac{4}{5}\cos 5x + \frac{3}{5}\sin 5x = \frac{6}{5}$

$\Rightarrow \quad \cos 5x \cos\alpha + \sin 5x \sin\alpha = \frac{6}{5} \quad \Rightarrow \quad \cos(5x - \alpha) = \frac{6}{5}$

Shunday α burchak mavjudki, $\cos\alpha = \frac{4}{5}$ va $\sin\alpha = \frac{3}{5}$ munosabatlar bo'ladigan, chunki $\cos^2\alpha + \sin^2\alpha = 1$ o'rinli.

Oxirgi tenglikning bo'lishi mumkin emas, chunki $-1 \le \cos(5x - \alpha) \le 1$ munosabatga zid.

Javob: B.

34. (07-120-24). Nechta butun son $\sin\left(\frac{16\pi}{x}\right) = 0$ tenglamani qanoatlantiradi?

A) 8 B) 10 C) 24 D) 16

Yechilishi. $\frac{16\pi}{x} = \pi n \qquad x = \frac{16}{n}, \ n \in Z$

n	1	-1	2	-2	4	-4	8	-8	16	-16
x	16	-16	8	-8	4	-4	2	-2	1	-1

Javob: B.

35. (07-132-24). $\sin^4 x - \cos^4 x = \frac{1}{2}$ tenglama $[-2\pi; 2\pi]$ kesmada nechta ildizga ega?

A) 9 B) 8 C) 7 D) 10

Yechilishi:

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \frac{1}{2} \Rightarrow -\cos 2x = \frac{1}{2} \Rightarrow \cos 2x = -\frac{1}{2} \Rightarrow$

$\Rightarrow \quad 2x = \pm\frac{2\pi}{3} + 2\pi n \quad \Rightarrow \quad x = \pm\frac{\pi}{3} + \pi n,\ n \in Z$

$\Rightarrow \quad x = \frac{\pi}{3} + \pi n,\ n \in Z$ *va* $x = -\frac{\pi}{3} + \pi k,\ k \in Z$ *hollarni qaraymiz*

n	0	1	-1	-2
x	$\frac{\pi}{3}$	$\frac{4\pi}{3}$	$-\frac{2\pi}{3}$	$-\frac{5\pi}{3}$

k	0	1	2	-1
x	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{3}$	$-\frac{4\pi}{3}$

Javob: B.

36. (07-137-24). $4\cos^2 2x - 2{,}5 = \cos 4x$ tenglamani yeching.

A) $\pm\frac{\pi}{12} + \frac{\pi n}{2},\ n \in Z$ B) $\frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z$

C) $\frac{\pi}{3} + \frac{\pi n}{2},\ n \in Z$ D) $\frac{\pi}{6} + \frac{\pi n}{2},\ n \in Z$

Yechilishi:

$\cos 4x = \cos(2 \cdot 2x) = \cos^2 2x - \sin^2 2x = \cos^2 2x - 1 + \cos^2 2x = 2\cos^2 2x - 1$

$4\cos^2 2x - \frac{5}{2} = 2\cos^2 2x - 1 \quad \Rightarrow \quad 2\cos^2 2x = \frac{3}{2} \quad \Rightarrow \quad \cos^2 2x = \frac{3}{4} \Rightarrow$

$\frac{1 + \cos 4x}{2} = \frac{3}{4} \Rightarrow \cos 4x = \frac{1}{2} \quad \Rightarrow \quad 4x = \pm\frac{\pi}{3} + 2\pi n \quad \Rightarrow \quad x = \pm\frac{\pi}{12} + \frac{\pi n}{2},\ n \in Z$

Javob: A

37. (07-151-24). k ning quyida ko'rsatilgan qiymatlaridan qaysi birida $\sin kx \cos x - \sin x \cos kx = 0$ tenglamaning ildizlari $\frac{\pi n}{7}\ (n \in Z)$ bo'ladi?

A) 8 B) 5 C) 7 D) 6

Yechilishi:

$\sin(kx - x) = 0 \quad \Rightarrow \quad x(k-1) = \pi n \quad \Rightarrow \quad x = \frac{\pi n}{k-1},\ k \in Z \quad \Rightarrow \quad k = 8$

Javob: A.

38. (07-153-24). Agar $2\sin 6x(\cos^4 3x-\sin^4 3x)=\sin kx$ tenglik hamma vaqt o'rinli bo'lsa, k ni toping.

A) 24 B)12 C) 18 D) 6

Yechilishi:

$2\sin 6x(\cos^2 3x-\sin^2 3x)(\cos^2 3x+\sin^2 3x)=\sin kx \Rightarrow \quad 2\sin 6x\cos 6x=\sin kx \quad \Rightarrow$

$\Rightarrow \quad \sin 12x=\sin kx \quad \Rightarrow \quad 12x=kx \quad \Rightarrow \quad k=12$

Javob: B.

39. (07-183-24). k ning quyida ko'rsatilgan qiymatlaridan qaysi birida $\cos kx\cos 4x-\sin kx\sin 4x=\dfrac{\sqrt{3}}{2}$ tenglamaning ildizlari $\pm\dfrac{\pi}{30}+\dfrac{2\pi n}{5}$ $n\in Z$ bo'ladi?

A) 3 B) 2 C) 1 D) 4

Yechilishi:

$$\cos(kx+4x)=\pm\frac{\sqrt{3}}{2},$$

$$(kx+4x)=\pm\frac{\pi}{6}+2\pi n \Rightarrow \quad x(k+4)=\pm\frac{\pi}{6}+2\pi n \Rightarrow$$

$$\Rightarrow \quad x=\pm\frac{\pi}{6(k+4)}+\frac{2\pi n}{(k+4)},\ k\in Z,\ k=1\ da$$

$$x=\pm\frac{\pi}{30}+\frac{2\pi n}{5}\quad o\text{'}rinli$$

Javob: C.

40. (08-103-13). $2\sin 3x-\sqrt{3}=0$ tenglamani yeching.

A) $(-1)^k\dfrac{\pi}{18}+\dfrac{\pi k}{3},\ k\in Z$ B) $(-1)^k\dfrac{\pi}{9}+\dfrac{\pi k}{3},\ k\in Z$

C) $(-1)^k\dfrac{\pi}{18}+\dfrac{2\pi k}{3},\ k\in Z$ D) $(-1)^k\dfrac{\pi}{9}+\dfrac{2\pi k}{3},\ k\in Z$

Yechilishi:

$$\sin 3x=\frac{\sqrt{3}}{2} \quad \Rightarrow \quad 3x=(-1)^k\frac{\pi}{3}+\pi k \quad \Rightarrow \quad x=(-1)^k\frac{\pi}{9}+\frac{\pi k}{3}, k\in Z$$

Javob: B.

41. (08-103-23). $\dfrac{\sin 2x}{\sin x+tgx}=0$ tenglamani yeching.

A) $\pi k,\ k\in Z$ B) $\dfrac{\pi}{2}+\pi k,\ k\in Z$

C) Ø D) $\dfrac{\pi k}{2},\ k\in Z$

Yechilishi: Masalani kasrni nolga tenglik shartidan va mantiqiy fikr yuritish bilan yechish maqsadga muvofiq.

$$\begin{cases}\sin 2x=0\\ \sin x\neq 0\\ \cos x\neq 0\end{cases} \Rightarrow \begin{cases}\sin x=0, \cos x=0\\ \sin x\neq 0\\ \cos x\neq 0\end{cases}$$

Oxirgi sistema ziddiyatli. Bundan tenglama yechimga ega emasligi kelib chiqadi.

Javob: C.

42. (08-103-35). $\dfrac{ctgx}{1+\sin x}=0$ tenglama $[0;5\pi]$ oraliqda necha ildizga ega?

A) 4 B) 5 C) 2 D) 3

Yechilishi:

Berilgan tenglama quyidagi sistemaga teng kuchli.

$$\begin{cases}ctgx=0\\ 1+\sin x\neq 0\end{cases} \Rightarrow \begin{cases}x=\dfrac{\pi}{2}+\pi k, k\in Z\\ x\neq -\dfrac{\pi}{2}+2\pi k, k\in Z\end{cases}$$

Endi berilgan kesmadagi ildizlar sonini aniqlaymiz.

n	0	1	2	3	4
x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$	$\frac{7\pi}{2}$	$\frac{9\pi}{2}$

$\frac{3\pi}{2}$ va $\frac{7\pi}{2}$ yechimlar kesmaga tegishli bo'lgani bilan chet ildizlardir, chunki maxrajni nolga aylantiryapti.

Javob: D.

TRIGONOMETRIK TENGSIZLIKLARNI YECHISH

Eng sodda trigonometrik tengsizliklar va ularning yechimlari quyidagi ko'rinishda bo'ladi:

$\sin x > a$ bu yerda $|a| < 1$, yechimi $x \in (\arcsin a + 2\pi n;\ \pi - \arcsin a + 2\pi n)$,

$\sin x < a$ bu yerda $|a| < 1$, yechimi $x \in (-\pi - \arcsin a + 2\pi n;\ \arcsin a + 2\pi n)$,

$\cos x > a$ bu yerda $|a| < 1$, yechimi $x \in (-\arccos a + 2\pi n;\ \arccos a + 2\pi n)$,

$\cos x < a$ bu yerda $|a| < 1$, yechimi $x \in (\arccos a + 2\pi n;\ 2\pi - \arccos a + 2\pi n)$,

$tgx > a$ yechimi $x \in (arctga + \pi n;\ \frac{\pi}{2} + \pi n)$,

$tgx < a$ yechimi $x \in (-\frac{\pi}{2} + \pi n;\ arctga + \pi n)$,

$ctgx > a$ yechimi $x \in (\pi n;\ arcctga + \pi n)$,

$ctgx < a$ yechimi $x \in (arcctga + \pi n;\ \pi + \pi n)$,

Barcha formulalarda $n \in Z$.

$y = \sin x$ funksiyaning eng kichik musbat davri 2π bo'lgani uchun

$$\sin x > a,\ \sin x \geq a,\ (1)$$

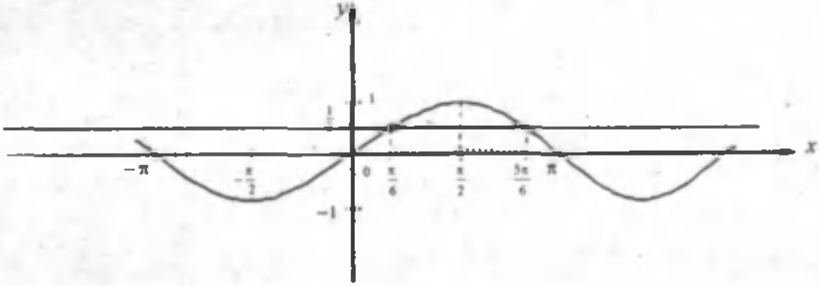
$$\sin x < a,\ \sin x \leq a,\ (2)$$

ko'rinishdagi tengsizliklarni istalgan 2π uzunlikka ega bo'lgan kesmada yechish yetarli. Barcha yechimlar to'plamini esa 2π uzunlikdagi kesmada topilgan har bir yechimga $2\pi n$, $n \in Z$ ni qo'yish bilan hosil qilinadi.

$\sin x > a$ va $\sin x \geq a$ ko'rinishdagi tengsizliklarni dastlab $\left[-\frac{\pi}{2}; \frac{3\pi}{2}\right]$ kesmada yechish qulay, $\sin x < a$ va $\sin x \leq a$ ko'rinishdagi tengsizliklarni esa dastlab $\left[\frac{\pi}{2}; \frac{5\pi}{2}\right]$ kesmada yechish qulaydir.

Misol. 1

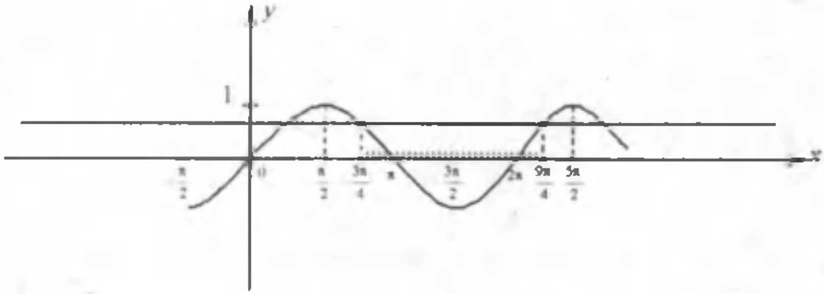
$\sin x > \frac{1}{2}$ tengsizlikni yeching.



$(-\frac{\pi}{2};\frac{\pi}{2})$ *oraliqda funksiya monoton o'suvchi*, u holda $\sin x = \frac{1}{2}$ tenglama bu oraliqda bitta $x = \frac{\pi}{6}$ ildizga ega bo'ladi, $(\frac{\pi}{2};\frac{3\pi}{2})$ oraliqda funksiya monoton kamayuvchi, demak bu oraliqda ham $\sin x = \frac{1}{2}$ tenglama yagona $x = \frac{5\pi}{6}$ ildizga egadir. $\frac{\pi}{6} < x < \frac{5\pi}{6}$ shartni qanoatlantiruvchi x ning barcha qiymatlari berilgan tengsizlikning yechimlari to'plamidan iborat bo'ladi. Sinusning davriyligidan foydalanib, tengsizlikning barcha yechimlari to'plami $(\frac{\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k)$, $k \in Z$ oraliqdan iborat bo'lishini topamiz.

2. $\sin x \le \frac{\sqrt{2}}{2}$ tengsizlikni yeching.

Yechish: Yuqorida ta'kidlanganidek, bu tengsizlikni dastlab $\left[\frac{\pi}{2};\frac{3\pi}{2}\right]$ kesmada yechimlarini izlaymiz.



Ko'rinib turibdiki, tengsizlik shartini qanoatlantiradigan x o'zgaruvchining barcha qiymatlari $\left[\frac{3\pi}{4}; \frac{9\pi}{4}\right]$ kesmadan iborat. U holda tengsizlikning yechimi:

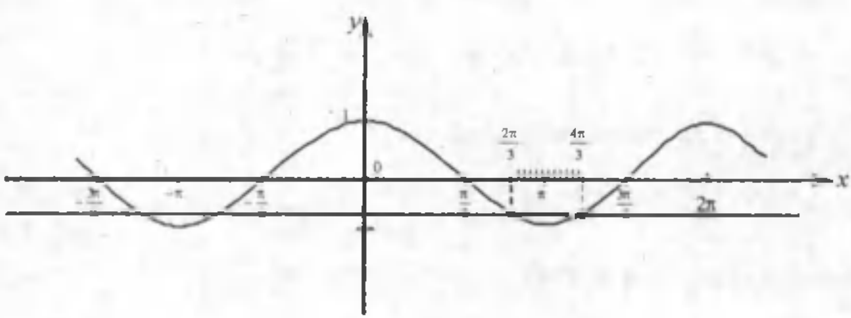
$$\frac{3\pi}{4} + 2\pi n \le x \le \frac{9\pi}{4} + 2\pi n, \; n \in Z \text{ bo'ladi.}$$

$\cos x < a$ va $\cos x \le a$ tengsizliklarning yechimlarini dastlab $[0; 2\pi]$ kesmada qarash kerak.

Misol.

$\cos x \le -\frac{1}{2}$ tengsizlikni yeching.

Yechish:



Tengsizlikning yechimlar to‘plami $y = -\frac{1}{2}$ to‘g‘ri chiziq bilan $y = \cos x$ funksiya grafigining kesishgan nuqtalaridan va kosinusoidaning $y = -\frac{1}{2}$ to‘g‘ri chiziqdan pastda joylashgan qismidan iborat.

Demak, $\frac{2\pi}{3} \le x \le \frac{4\pi}{3}$ ga $2\pi n$ davrni qo‘shib,

$$\frac{2\pi}{3} + 2\pi n \le x \le \frac{4\pi}{3} + 2\pi n, \ n \in Z$$

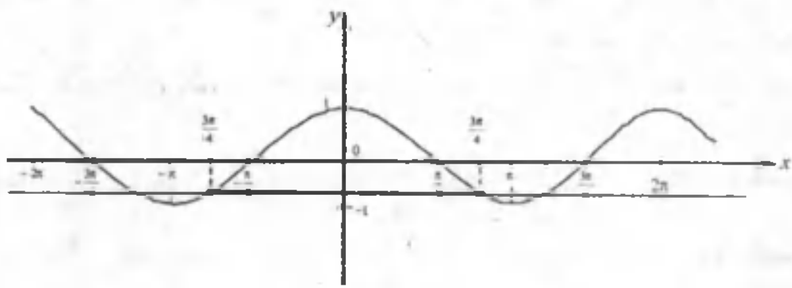
javobni hosil qilamiz.

$\cos x > a$ va $\cos x \ge a$ ko‘rinishdagi tengsizliklarni yechishda yechimni dastlab $[-\pi; \pi]$ kesmadan qidirgan ma'qul.

Misol.

$\cos x > -\frac{\sqrt{2}}{2}$ tenglamani yeching.

Yechish:



Shakldan $[-\pi; \pi]$ kesmadagi yechimlar to‘plami $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$ ekanini topamiz. U holda berilgan tengsizlikning barcha yechimlar to‘plami

$$-\frac{3\pi}{4} + 2\pi n < x < \frac{3\pi}{4} + 2\pi n, \quad n \in Z$$

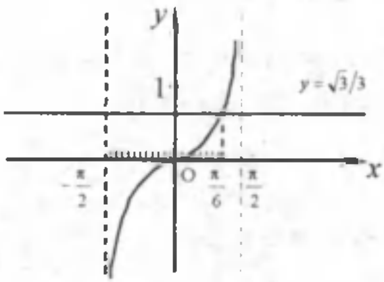
ko'rinishda bo'ladi.

$tgx > a,\ tgx \geq a,\ tgx < a,\ tgx \leq a$ ko'rinishdagi tengsizliklarning yechimini dastlab $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda axtargan ma'qul.

Misol.

$tgx < \frac{\sqrt{3}}{3}$ tengsizlikni yeching.

Yechish:



Shaklda berilgan tengsizlikning yechimi $y = \frac{\sqrt{3}}{3}$ to'g'ri chiziqdan pastki qismi ekani ko'rinib turibdi.

$y = tgx$ funksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda monoton o'suvchi bo'lgani uchun bu oraliqda $tgx = \frac{\sqrt{3}}{3}$ tenglamaning yagona ildizi mavjud, u ham $x = \frac{\pi}{6}$ nuqtadir. U holda berilgan tengsizlikning $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqdagi yechimlari $-\frac{\pi}{2} < x < \frac{\pi}{6}$ bo'lib, barcha yechimlar to'plami $-\frac{\pi}{2} + \pi k < x < \frac{\pi}{6} + \pi k,\ \ k \in Z$ ko'rinishda bo'ladi.

$ctgx > a,\ ctgx \geq a,\ ctgx < a,\ ctgx \leq a$ ko'rinishdagi tengsizliklarning yechimlarini $(0;\pi)$ oraliqdan izlagan ma'qul.

tgx va $ctgx$ bilan bog'liq tengsizliklarni yechganda ularning eng kichik musbat davri π ekanini va izlangan intervaldan aniqlangan yechimga πn ni qo'shish bilan yuqoridagi tengsizliklarning barcha yechimlarini topish mumkinligini esdan chiqarmaslik kerak.

Trigonometrik tengsizliklarni yechishda ham tengsizlikning bir xil nomdagi trigonometrik funksiyalar qatnashgan tengsizlikka keltirib, yordamchi o'zgaruvchi kiritish yoki ko'paytuvchilarga ajratish usullaridan foydalanish mumkin.

Misol. 1.

$2\sin^2 x - 7\sin x + 3 > 0$ tengsizlikni yeching.

Yechish: Ushbu tengsizlikda $\sin x = y$ deb belgilab, $2y^2 - 7y + 3 > 0$ tengsizlikni yechib, yechimlar to'plami $y < \frac{1}{2}$ va $y > 3$ bo'lishini topamiz.

$y > 3$ da $\sin x > 3$ bo'lgani uchun,bu holda tengsizlik yechimga ega emas.

$y < \frac{1}{2}$ bo'lgan holda esa $\sin x < \frac{1}{2}$ bo'lib, uning yechimlar to'plami:

$$\left(-\frac{7\pi}{6} + 2\pi n,\ \frac{\pi}{6} + 2\pi n\right),\ n \in Z$$

oraliqdan iborat bo'ladi. Bu berilgan tengsizlikning yechimlar to'plamidir.

2. $\cos x + \cos 2x + \cos 3x > 0$ tengsizlikni yeching.

Yechish:

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$ formuladan

foydalanib, $\cos x + \cos 3x$ yig'indini shakl almashtirib olamiz.

$\cos x + \cos 3x = 2\cos 2x \cdot \cos x > 0$

Buni berilgan tengsizlikka qo'yib, $\cos 2x + 2\cos 2x \cdot \cos x > 0$ tengsizlikni hosil qilamiz. Oxirgi tengsizlikni ko'paytuvchilarga ajratib yechamiz.

$$\cos 2x(1 + 2\cos x) > 0$$, bundan

$$\begin{cases}\cos 2x > 0,\\ 1 + 2\cos x > 0,\end{cases} \text{ yoki } \begin{cases}\cos 2x < 0,\\ 1 + 2\cos x < 0.\end{cases}$$

tengsizliklar sistemalari kelib chiqadi. Bularni yechib,

$$\left(\frac{2\pi}{3}+2\pi n;\ \frac{3\pi}{4}+2\pi n\right)\cup\left(\frac{5\pi}{4}+2\pi n;\ \frac{4\pi}{3}+2\pi n\right)\cup\left(-\frac{\pi}{4}+2\pi n;\ \frac{\pi}{4}+2\pi n\right),\ n\in Z$$

javobni topamiz.

Teskari trigonometrik funksiyalar qatnashgan eng sodda trigonometrik tengsizliklar va ularning yechimlari to'plami quyidagilardan iborat:

$$\arcsin x > a,\ \left(|a| < \frac{\pi}{2}\right) \text{ yechimi } x \in (\sin a; 1]$$

$$\arcsin x < a,\ \left(|a| \le \frac{\pi}{2}\right) \text{ yechimi } x \in [-1; \sin a)$$

$$\arccos x > a,\ (0 < a < \pi) \text{ yechimi } x \in [-1; \cos a)$$

$$\arccos x < a,\ (0 < a \le \pi) \text{ yechimi } x \in (\cos a; 1]$$

$$arctgx > a,\ \left(|a| < \frac{\pi}{2}\right) \text{ yechimi } x \in (tga; +\infty)$$

$$arctgx < a,\ \left(|a| < \frac{\pi}{2}\right) \text{ yechimi } x \in (-\infty; tga)$$

$$arcctgx > a,\ (0 < a < \pi) \text{ yechimi } x \in (-\infty; ctga)$$

$$arcctgx < a,\ (0 < a < \pi) \text{ yechimi } x \in (ctga; +\infty)$$

TRIGONOMETRIK TENGSIZLIKLARGA DOIR MASALALAR YECHISH

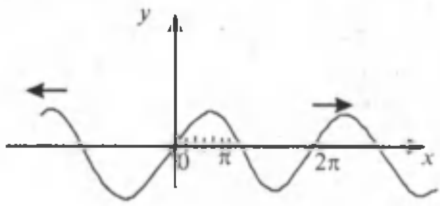
1. (96-13-34). Ushbu $y=\sqrt{1+\log_{\frac{1}{2}}\sin x}$ funksiya $x\left(x\in[0;2\pi]\right)$ ning qanday qiymatlarida aniqlangan?

$A)\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$ $B)(0;\frac{\pi}{6}]\cup[\frac{5\pi}{6};\pi)$ $C)(0;\frac{\pi}{6}]$ $D)(0;\pi)$ $E)[\frac{5\pi}{6};\pi)$

Yechilishi.

$$\begin{cases}1+\log_{\frac{1}{2}}\sin x\ge 0\\ \sin x>0\\ 0\le x\le 2\pi\end{cases}\Leftrightarrow\begin{cases}\log_{\frac{1}{2}}\sin x\ge -1\\ \sin x>0\\ 0\le x\le 2\pi\end{cases}\Rightarrow\begin{cases}\sin x\le 2\\ \sin x>0\\ 0\le x\le 2\pi\end{cases}\Rightarrow\begin{cases}x\in(-\infty;\infty)\\ \sin x>0\\ 0\le x\le 2\pi\end{cases}$$

$$\Rightarrow\begin{cases}x\in(-\infty;\infty)\\ 2\pi n<x<\pi+2\pi n,\ n\in Z\Rightarrow x\in(0;\pi)\\ 0\le x\le 2\pi\end{cases}$$



$x\in(0;\pi)$

Javob: D.

2. (97-2-44). m ning qanday qiymatlarida $y=\cos x+mx$ funksiya aniqlanish sohasida kamayadi?

$A)m\in(-\infty;-1]$ $B)m\in(-1;\infty)$ $C)m\in[-1;\infty)$ $D)m\in(-\infty;1]$ $E)m\in[-1;1]$

Yechilishi. $y'\le 0$ tengsizlikni qanoatlantiruvchi barcha x larda berilgan funksiya kamayuvchi bo'ladi.

$-\sin x+m\le 0 \quad\Rightarrow\quad -\sin x\le -m\ \Rightarrow\quad \sin x\ge m$

$y=\sin x$ funksiya $[-1;1]$ kesmada chegaralanganligidan oxirgi tengsizlik $m\in(-\infty;-1]$ oraliqda o'rinli ekanligi kelib chiqadi

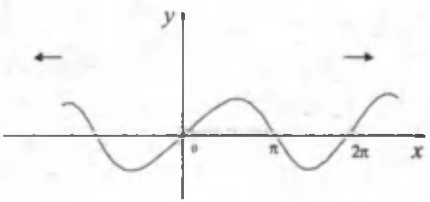
Javob: A.

3. (97 – 9 – 38). Ushbu $y=\log_5(5\sin x)$ funksiyaning aniqlanish sohasini toping

$A)\left(-\pi+2\pi n;\frac{\pi}{2}+2\pi n\right), n\in Z$ $\quad B)(2\pi n;\pi+2\pi n), n\in Z$ $\quad C)\left(-\pi n;\frac{\pi}{2}+2\pi n\right), n\in Z$

$D)\left(\pi n;\frac{3\pi}{4}+2\pi n\right), n\in Z$ $\quad E)\left(\pi n;\frac{\pi}{2}+2\pi n\right), n\in Z$

Yechilishi. Logarifmik funksiyaning aniqlanish sohasidan $5\sin x>0$ tengsizlikni yoza olamiz. $\sin x>0$



$0+2\pi n<x<\pi+2\pi n$ *yoki*

$(2\pi n;\ \pi+2\pi n), n\in Z$

Javob: B.

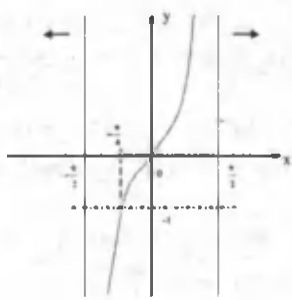
4. (97 – 11 – 47). Ushbu $y=\sqrt{tgx+1}$ funksiyaning aniqlanish sohasini toping.

$A)\left[-\frac{\pi}{4}+\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$ $\quad B)\left[\frac{\pi}{4}+\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$ $\quad C)\left[-\frac{\pi}{4}+\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$

$D)\left[-\frac{\pi}{2}+\pi n;-\frac{\pi}{4}+\pi n\right], n\in Z$ $\quad E)\left[-\frac{\pi}{2}+\pi n;\frac{\pi}{4}+\pi n\right], n\in Z$

Yechilishi:

Kvadrat ildiz ostidagi ifoda nomanfiy ekanligidan ushbu $tgx+1\geq 0$ tengsizlikni yozish mumkin.



$tgx+1\geq 0$

$tgx\geq -1$

$-\frac{\pi}{4}+\pi n\leq x<\frac{\pi}{2}+\pi n$

yoki $[-\frac{\pi}{4}+\pi n;\ \frac{\pi}{2}+\pi n),\ n\in Z$ (*grafikka qarang*)

Javob: C.

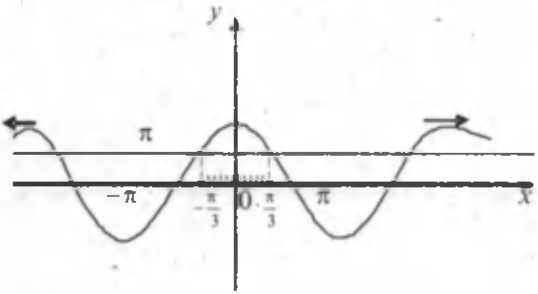
5. (98 – 9 – 24). x ning $(-\pi;\pi)$ oraliqqa tegishli qanday qiymatlarida $|\cos x+2{,}5|\geq 3$ tengsizlik o'rinli bo'ladi?

$A)\left[-\frac{\pi}{3};\frac{\pi}{3}\right]$ $B)\left(-\frac{\pi}{6};\frac{\pi}{6}\right)$ $C)\left(-\frac{\pi}{3};\frac{\pi}{3}\right)$ $D)\left[-\frac{\pi}{6};\frac{\pi}{6}\right]$ $E)\left[-\frac{\pi}{4};\frac{\pi}{4}\right]$

Yechilishi.

$$\begin{cases}\cos x+2{,}5\geq 3\\ -\pi<x<\pi\end{cases}\cup\begin{cases}\cos x+2{,}5\leq -3\\ -\pi<x<\pi\end{cases}\Rightarrow\begin{cases}\cos x\geq\frac{1}{2}\\ -\pi<x<\pi\end{cases}$$

Oxirgi sistemaning yechimini grafikdan foydalanib hosil qilamiz



$x\in\left[-\frac{\pi}{3};\frac{\pi}{3}\right]$

(Chizmaga qarang)

Javob: A.

6. (99-4-56). Tengsizlikni yeching

$$\cos 4\cdot\cos x\geq\sqrt{\frac{\cos x}{1+ctg^2x}}$$

$A)(\pi n;\frac{\pi}{2}+\pi n],n\in Z$ $B)\left[0;\frac{\pi}{2}\right]$ $C)\frac{\pi}{4}+\pi n,n\in Z$ $D)\pi n,n\in Z$

$E)\left[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right],n\in Z$

Yechilishi:

$\cos 4<0$ ekanligi ravshan. $\cos x<0$ bo'la olmaydi, chunki $\frac{\cos x}{1+ctg^2x}<0$ bo'ladi, buning esa bo'lishi mumkin emas. Faqat $\cos x=0$ bajariladi.

$$x = \frac{\pi}{2} + \pi n; n \in Z$$

Javob: C.

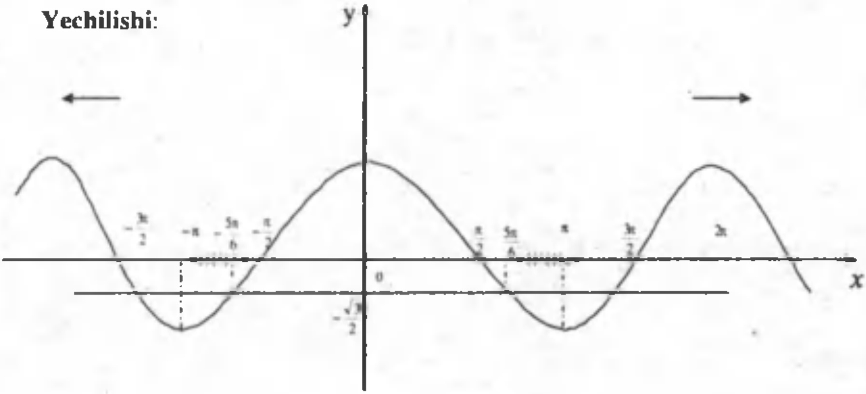
7. (00 – 3 – 55). Quyidagi tengsizlik

$$-1 - \frac{2}{\sqrt{3}} \cos x > 0$$

$[-\pi; \pi]$ kesmada nechta butun yechimga ega?

A) 4 B) 3 C) 6 D) 5 E) 2

Yechilishi:



$$-1 - \frac{2}{\sqrt{3}} \cos x > 0$$

$$-\frac{2}{\sqrt{3}} \cos x > 1$$

$$\cos x < -\frac{\sqrt{3}}{2}$$

$$\left[-\pi; -\frac{5\pi}{6}\right) \cup \left(\frac{5\pi}{6}; \pi\right]$$

$\pi \approx 3{,}14$ *va* $\frac{5\pi}{6} \approx 2{,}6$ ekaniga e'tibor bersak, bu oraliqda -3 va 3 butun sonlari mavjudligiga ishonch hosil qilish mumkin.

Javob: E.

8. (00 – 6 – 56). Tengsizlikni yeching

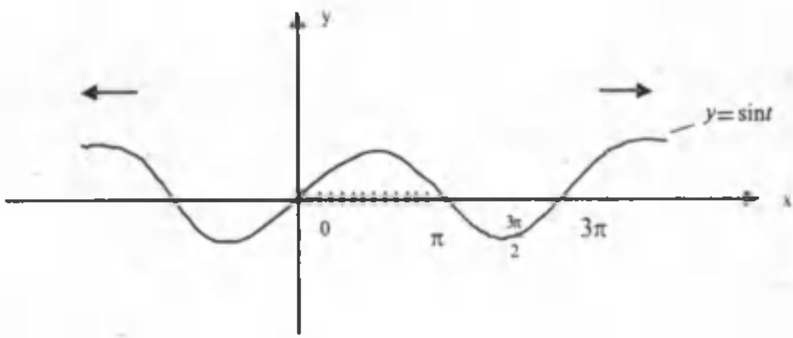
$$\cos x < \sin x$$

A) $\left(\frac{\pi}{4}+\pi k;\frac{3\pi}{4}+\pi k\right), k\in Z.$ B) $\left(\frac{\pi}{4}+\pi k;\frac{5\pi}{4}+\pi k\right), k\in Z.$ C) $\left(\frac{\pi}{4}+2\pi k;\frac{3\pi}{4}+2\pi k\right), k\in Z.$

D) $(2\pi k;\pi+2\pi k), k\in Z.$ E) $\left(\frac{\pi}{4}+2\pi k;\frac{5\pi}{4}+2\pi k\right), k\in Z.$

Yechilishi:

Yordamchi burchak kiritish usuli bilan soddalashtiramiz va grafik yordamida yechimini olamiz.



$$\frac{\sqrt{2}}{2}\cos x < \frac{\sqrt{2}}{2}\sin x$$

$$\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}>0.$$

$\sin\left(x-\frac{\pi}{4}\right)>0,\quad x-\frac{\pi}{4}=$ t desak, ushbu $0+2\pi n < t < \pi+2\pi n$, o'rinli.

Endi belgilashga qaytamiz

$$0+2\pi n<x-\frac{\pi}{4}<\pi+2\pi n$$

$$\frac{\pi}{4}+2\pi n<x<\frac{5\pi}{4}+2\pi n \qquad yoki \qquad \left(\frac{\pi}{4}+2\pi n;\ \frac{5\pi}{4}+2\pi n\right),\ n\in Z$$

Javob: E.

9.(00-9-28). Tengsizlikni yeching.

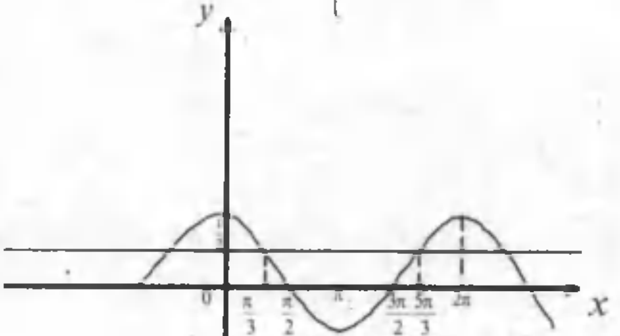
$$\left(\frac{\pi}{2}-\frac{e}{3}\right)^{\ln(2\cos x)}\geq 1, \qquad (x\in[0;2\pi])$$

A) $[\frac{\pi}{3};\frac{\pi}{2}]\cup[\frac{3\pi}{2};\frac{5\pi}{3}]$ B) $[\frac{\pi}{3};\frac{5\pi}{3}]$ C) $[\frac{\pi}{3};\frac{\pi}{2}]$

D) $[\frac{\pi}{6};\frac{\pi}{2}]\cup[\frac{3\pi}{2};\frac{5\pi}{6}]$ E) $[\frac{\pi}{3};\frac{\pi}{2}]\cup[\frac{3\pi}{2};\frac{5\pi}{3}]$

Yechilishi. $0<\frac{\pi}{2}-\frac{e}{3}<1$ va $2\cos x>0$ ekanligiga e'tibor bergan holda ushbu sistemaga ega bo'lamiz.

$$\begin{cases}\left(\frac{\pi}{2}-\frac{e}{3}\right)^{\ln(2\cos x)}\geq\left(\frac{\pi}{2}-\frac{e}{3}\right)^{0}\\ 2\cos x>0\\ 0\leq x\leq 2\pi\end{cases} \Leftrightarrow \begin{cases}\cos x\leq\frac{1}{2}\\ \cos x>0\\ 0\leq x\leq 2\pi\end{cases}$$



$$[\frac{\pi}{3};\frac{\pi}{2})\cup(\frac{3\pi}{2};\frac{5\pi}{3}]$$

Javob: E.

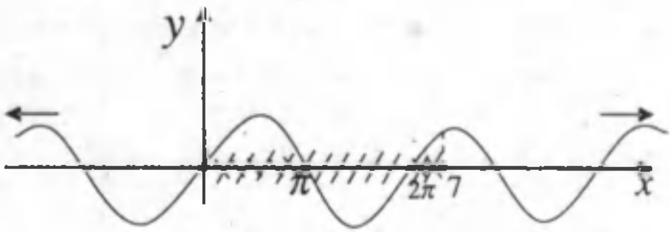
10.(00-10-63). Funksiyaning aniqlanish sohasini toping

$$y=\lg\sin x+\sqrt{-x^2+7x}$$

$A)\,(0;\pi)\cup(2\pi;7]$ $B)\,(-1;1)$ $C)\,[0;7]$ $D)\,[0,\pi]$ $E)\,(0,\pi)\cup(\pi;2\pi)$

Yechilishi: Funksiyaning aniqlanish sohasi ushbu $\begin{cases}\sin x>0\\ -x^2+7x\ge 0\end{cases}$ sistemadan kelib chiqadi

$$\begin{cases}\sin x>0\\ x(x-7)\le 0\end{cases}\Rightarrow\begin{cases}\sin x>0\\ 0\le x\le 7\end{cases}\Rightarrow[0;\pi)\cup(2\pi;7]$$



Javob: A.

11. (01-2-79). $[-13;18]$ kesmadagi nechta butun son $y=\sqrt{|x|+x}+\sqrt{\sin^2(\pi x)}$ funksiyaning aniqlanish sohasiga tegishli?

A) 31   B) 32   C) 22   D) 63   E) 24

Yechilishi:

Kvadrat ildiz ostidagi ifoda nomanfiy ekanligidan ushbu $\begin{cases}|x|-x\ge 0\\ -\sin^2(\pi x)\ge 0\end{cases}$ sistema o'rinli bo'ladi. U holda masala shartiga ko'ra quyidagilar o'rinli.

$$\begin{cases} -13 \le x \le 18 \\ |x| - x \ge 0 \\ -\sin^2(2\pi x) \ge 0 \end{cases} \Rightarrow \begin{cases} -13 \le x \le 18 \\ x \in (-\infty; \infty) \\ 2\pi x = \pi n \end{cases} \Rightarrow \begin{cases} -13 \le x \le 18 \\ x \in (-\infty; \infty) \\ x = \dfrac{n}{2}, n \in Z \end{cases}$$

Bundan n= -26, -24, -22, ..., 0, 2, 4, ..., 36 larda x ning 32 ta butun yechimi hosil bo'ladi.

Javob:B.

12. (01-4-2). $[0; 2\pi]$ kesmaga tegishli nechta nuqta $y = \ln\left(2\sin 3x + 3\cos 2x - \dfrac{17}{3}\right)$ finksiyaning aniqlanish sohasiga tegishli?

A) Ø B) 1 C) 2 D) 3 E) 4

Yechilishi: Logarifmik funksiyaning aniqlanish sohasiga ko'ra $2\sin 3x + 3\cos 2x - \dfrac{17}{3} > 0$. Endi $2\sin 3x + 3\cos 2x < 5$ ekanidan $2\sin 3x + 3\cos 2x - \dfrac{17}{3} > 0$ tengsizlikning bajarilmasligi kelib chiqadi.

Javob: A.

13. (01-4-4). Ushbu $\arccos^2 x - \dfrac{5\pi}{6} \cdot \arccos x + \dfrac{\pi^2}{6} \le 0$ tengsizlik o'rinli bo'ladigan kesmaning o'rtasini toping.

$A)\, 0{,}5$ $B)\, 0{,}4$ $C)\, 0{,}25$ $D)\, \dfrac{\pi}{4}$ $E)\, \dfrac{\pi}{2}$

Yechilishi: $\arccos x = t$ deb belgilash kiritamiz.

$t^2 - \dfrac{5\pi}{6} t + \dfrac{\pi^2}{6} \le 0 \Rightarrow 6t^2 - 5\pi t + \pi^2 \le 0$. Bu tengsizliklarni yechib, $\dfrac{\pi}{6} \le t \le \dfrac{\pi}{2}$ ga ega bo'lamiz

$$\frac{\pi}{6} \le \arccos x \le \frac{\pi}{2} \quad \Rightarrow \quad \frac{1}{2} \ge x \ge 0 \qquad ya'ni \quad x \in \left[0; \frac{1}{2}\right]$$

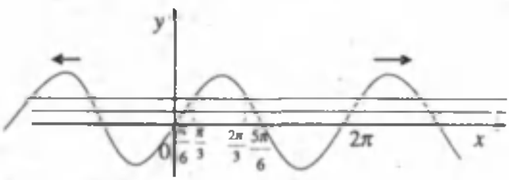
Endi bu kesmaning o'rtasi 0,25 ga tengligini payqash qiyin emas.

Javob: C.

14. (01-11-22). Ushbu $2^{\frac{1}{2}} \le 2^{\sin x} \le 2^{\frac{\sqrt{3}}{2}}$ tengsizlikning $[0; 2\pi]$ oraliqdagi eng katta va eng kichik yechimlari yig'indisini toping.

$A)\ \frac{2\pi}{3}$ $\quad B)\ \pi$ $\quad C)\ \frac{4\pi}{6}$ $\quad D)\ \frac{\pi}{2}$ $\quad E)\ \frac{3\pi}{4}$

Yechilishi: Berilgan tengsizlik $\frac{1}{2} \le \sin x \le \frac{\sqrt{3}}{2}$ tengsizlikka teng kuchli. Endi masala shartining javobini grafikdan foydalanib topamiz.



$x \in \left[\frac{\pi}{6}; \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}; \frac{5\pi}{6}\right]$.

Bu oraliqdagi eng katta yechim $\frac{5\pi}{6}$ va eng kichik yechim $\frac{\pi}{6}$.

$$\frac{5\pi}{6} + \frac{\pi}{6} = \pi$$

Javob: B.

15. (01-12-27). Tengsizlikni yeching $\lg(\arcsin x) > -1$.

$A)\ (0; \frac{\pi}{2}]$ $\quad B)\ [\sin -,1; 1]$ $\quad C)\ (\sin 0,1; 1)$ $\quad D)\ (\sin 0,1; 1]$ $\quad E)\ \varnothing$

Yechilishi: Tengsizlik ushbu sistemaga teng kuchli bo'ladi:

$$\begin{cases} \arcsin x > 0 \\ -1 \le x \le 1 \\ \arcsin x > 0,1 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ -1 \le x \le 1 \\ x > \sin 0,1 \end{cases} \Rightarrow x \in (\sin 0,1; 1]$$

Javob: D.

16.(02-1-62). $\cos(\sin x) < 0$ tengsizlikni yeching.

$A)\left(\frac{\pi}{2}+2\pi n; \frac{3\pi}{2}+2\pi n\right),\ n\in Z$ $B)\left(\frac{\pi}{2}+\pi n; \frac{3\pi}{2}+\pi n\right),\ n\in Z$ $C)\left(0; \frac{3\pi}{2}+2\pi n\right),\ n\in Z$

$D)\left(0; \frac{3\pi}{2}\right),\ n\in Z$ *E) yechimga ega emas*

Yechilishi: $-1\le \sin x\le 1$ va $y=\cos x$ juft ekanligiga e'tibor bersak, masala $y=\cos x$ funksiyaning 0 radiandan 1 radiangacha bo'lgan barcha qiymatlarini topishga keltirildi. $y=\cos x$ funksiya bu holdagi qiymatlari esa musbat. Ziddiyatga uchradik.

Demak tengsizlik yechimga ega emas.

Javob: E.

17. (02-10-62). $\cos^2 x-\cos x+\frac{1}{4}\ge\frac{1}{2}$ tengsizlikni yeching.

$A)\left[\frac{\pi}{2}+2\pi n; \frac{3\pi}{2}+2\pi n\right]\cup\{2\pi n\},n\in Z$ $B)\left[-\frac{\pi}{2}+2\pi n; \frac{\pi}{2}+2\pi n\right]\cup\{2\pi n\},n\in Z$

$C)\left[-\frac{\pi}{2}+2\pi n; \pi+2\pi n\right]\cup\{2\pi n\},n\in Z$ $D)\left[\frac{5\pi}{6}+2\pi n; \frac{4\pi}{3}+2\pi n\right],n\in Z$

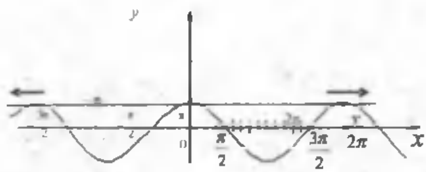
Yechilishi: Berilgan tengsizlikni kvadratga oshirib yechamiz.

$\cos^2 x-\cos x\ge 0$

$\cos x(\cos x-1)\ge 0$

$\cos x\le 0,\ \cos x\ge 1$ *va chizmadan*

$\left[\frac{\pi}{2}+2\pi n; \frac{3\pi}{2}+2\pi n\right]\cup\{2\pi n\},n\in Z$



Javob: A.

18. (03-1-18). $\sin x<1+\frac{x^2}{4}$ tengsizlikni yeching.

$A)\ \varnothing$ $B)\left(-\frac{\pi}{2}+2\pi n; \frac{\pi}{2}+2\pi n\right),\ n\in Z$ $C)[-\pi;\pi]$ $D)\left(-\frac{\pi}{6}+2\pi n; \frac{\pi}{6}+2\pi n\right),\ n\in Z$

$E)(-\infty;\infty)$

Yechilishi. Tengsizlikning o'ng tomoni $x=0$ da eng kichik qiymatga erishadi va berilgan tengsizlik bajariladi. x ning qolgan barcha qiymatlarida ham tengsizlik o'rinli bo'ladi, chunki $E(\sin x)=[-1;1]$

Javob: E.

19. (03-1-36). $\frac{5^{x^2}-5}{3\sin x+4\cos x-2\pi}\geq 0$ tengsizlikni yeching.

$A)\ [-1;1]$ $B)\ \left[1;\frac{\pi}{2}\right]$ $C)\ [-1;\pi]$ $D)\ [0;\pi]$ $E)\ [1;\pi]$

Yechilishi: $3\sin x+4\cos x$ ifodaning eng katta qiymati 5 ga, eng kichik qiymati -5 ga teng ($E(a\sin kx+b\cos kx)=\left[-\sqrt{a^2+b^2};\sqrt{a^2+b^2}\right]$ ekanligidan). Bundan ko'rinadiki maxraj har doim manfiy. Kasr ifoda nomanfiy qiymat qabul qilishi uchun uning surati nomusbat bo'lishi kerak, ya'ni

$$5^{x^2}-5\leq 0$$

$$5^{x^2}\leq 5^1$$

$$x^2\leq 1$$

$$x\in[-1;1]$$

Javob: A.

20. (03-1-54). $\left(\cos x+\frac{\pi}{2}\right)\cdot\left(\sin x-\frac{\pi}{3}\right)\cdot\left(tg^2x-\frac{1}{3}\right)\geq 0$ tengsizlikni yeching.

$A)\left[-\frac{\pi}{3}+\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$ $B)\left[-\frac{\pi}{3}+\pi n;\frac{\pi}{3}+\pi n\right], n\in Z$ $C)\left[-\frac{\pi}{3}+\pi n;\frac{\pi}{6}+\pi n\right], n\in Z$

$D)\left[-\frac{\pi}{6}+\pi n;\frac{\pi}{3}+\pi n\right], n\in Z$ $E)\left[-\frac{\pi}{6}+\pi n;\frac{\pi}{6}+\pi n\right], n\in Z$

Yechilishi:

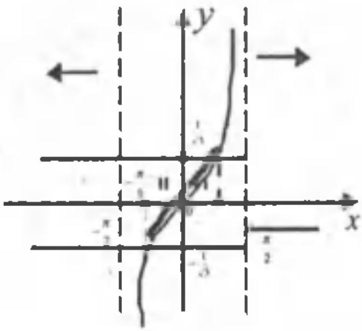
$$\left(\cos x+\frac{\pi}{2}\right)\cdot\left(\sin x-\frac{\pi}{3}\right)<0,\ tg^2x-\frac{1}{3}\leq 0$$

bo'lgani uchun, tengsizlikning ikkala qismiga bo'lamiz. Chunki,

$\cos x + \frac{\pi}{2} > 0$, $\sin x - \frac{\pi}{3} \le 0$) $tg^2 x - \frac{1}{3} \le 0$

$$\left(tgx - \frac{1}{\sqrt{3}}\right)\left(tgx + \frac{1}{\sqrt{3}}\right) \le 0 \quad \Leftrightarrow \quad -\frac{1}{\sqrt{3}} \le tgx \le \frac{1}{\sqrt{3}}$$

Endi yechimni grafik yordamida topamiz:



$$\left[-\frac{\pi}{6} + \pi n; \frac{\pi}{6} + \pi n\right], n \in Z$$

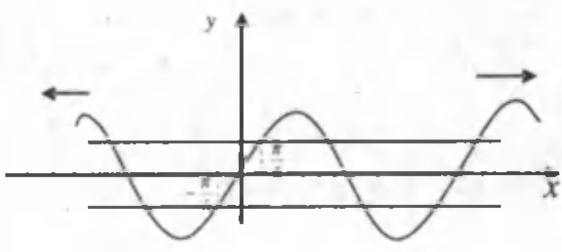
Javob: E.

21. (03-2-31). $\cos(\pi \sin x) > 0$ tengsizlikni yeching

A) $\left(\pi k; \frac{\pi}{3} + \pi k\right), k \in Z$ B) $\left(-\frac{\pi}{6} + \pi k; \frac{\pi}{6} + \pi k\right), k \in Z$ C) $\left(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k\right), k \in Z$

D) $\left(\pi k; \frac{\pi}{6} + \pi k\right), k \in Z$ E) $\left(-\frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k\right), k \in Z$

Yechilishi. $\cos(\pi \sin x) > 0$



Oldin

$$-\frac{\pi}{2} < \pi \sin x < \frac{\pi}{2}$$

tengsizlikni hosil qilamiz. Uni quyidagicha yechamiz.

$$-\frac{1}{2} < \sin x < \frac{1}{2}$$

$$x \in \left(-\frac{\pi}{6} + \pi n; \frac{\pi}{6} + \pi n\right), n \in Z$$

Javob: B.

22. (03-4-27). $1 \le \frac{tg3x + tgx}{1 - tg3xtgx} \le \sqrt{3}$ $(0 < x < \pi)$ tengsizlikning eng katta va eng kichik yechimlari yig'indisini toping.

$A)\frac{\pi}{7}$ $B)\frac{43\pi}{48}$ $C)\frac{5\pi}{48}$ $D)\frac{7\pi}{48}$ $E)\frac{3\pi}{16}$

Yechilishi:

$$1 \le tg4x \le \sqrt{3}$$

$$\frac{\pi}{4} + \pi n \le 4x \le \frac{\pi}{3} + \pi n$$

$$\frac{\pi}{16} + \frac{\pi n}{4} \le x \le \frac{\pi}{12} + \frac{\pi n}{4}$$

n=0 da $\frac{\pi}{16}$ eng kichik yechim, *n=3* da $\frac{5\pi}{6}$ eng katta yechimni hosil qilamiz. Endi masala shartiga ko'ra,

$$\frac{\pi}{16} + \frac{5\pi}{6} = \frac{43\pi}{48}$$

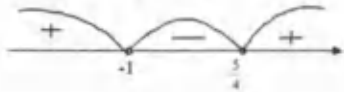
Javob: B.

23. (03-11-27). *a* parametrning qanday qiymatlarida $\sin x \le \frac{3a-6}{a+1}$ tengsizlik yechimga ega emas?

$A)\left(-1; \frac{5}{4}\right)$ $B)(-1;0)$ $C)(-1;2)$ $D)(-1;5)$ $E)(0;\infty)$

Yechilishi: $-1 \le \sin x \le 1$ ekanligidan $\sin x \le \frac{3a-6}{a+1}$ tengsizlik yechimga ega bo'lmasligi uchun $\frac{3a-6}{a+1} < -1$ tengsizlik bajarilishi shart.

$$\frac{3a-6}{a+1}+1<0 \Rightarrow \frac{4a-5}{a+1}<0 \Rightarrow \frac{a-\frac{5}{4}}{a+1}<0$$



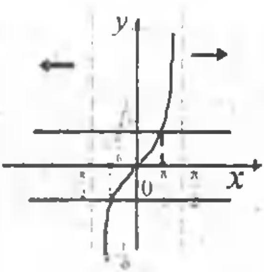
$$a \in \left(-1;\frac{5}{4}\right)$$

Javob: A.

24. (03-12-62). $\left(-2x^2+5x-7\right)\left(3tg^2x-1\right)\geq 0$ tengsizlikni yeching.

A) yechimga ega emas $B)\left[-\frac{\pi}{6}+\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$ $C)\left[-\frac{\pi}{2}+\pi n;\frac{\pi}{6}+\pi n\right], n\in Z$

$D)\left[-\frac{\pi}{6}+\pi n;\frac{\pi}{6}+\pi n\right], n\in Z$ $E)(-\infty;\infty)$



Yechilishi: $-2x^2+5x-7<0$ ekanidan $(D<0, a<0)$ tengsizlikning ikkala qismini $-2x^2+5x-7$ ga bo'lamiz. Natijada ushbu $3tg^2x-1\leq 0$ tengsizlik hosil bo'ladi.

$$\left(tgx-\frac{1}{\sqrt{3}}\right)\left(tgx+\frac{1}{\sqrt{3}}\right)\leq 0$$

$$-\frac{1}{\sqrt{3}}\leq tgx\leq\frac{1}{\sqrt{3}} \Leftrightarrow x\in\left[-\frac{\pi}{6}+\pi n;\frac{\pi}{6}+\pi n\right], n\in Z$$

Javob: D.

25. (05-114-32). $\sin^2 x-\frac{5}{2}\sin x+1>0$ tengsizlik $x\left(x\in[0;2\pi]\right)$ ning qanday qiymatlarida o'rinli bo'ladi?

A) $\left(0;\frac{\pi}{3}\right)\cup\left(\frac{2\pi}{3};2\pi\right]$ B) Ø C) $\left[0;\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6};2\pi\right]$

D) $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3};2\pi\right]$ E) $\left(\frac{\pi}{6};\frac{5\pi}{6}\right)$

Yechilishi: $\sin x = a$ deb belgilab olamiz

$a^2-\frac{5}{2}a+1>0$

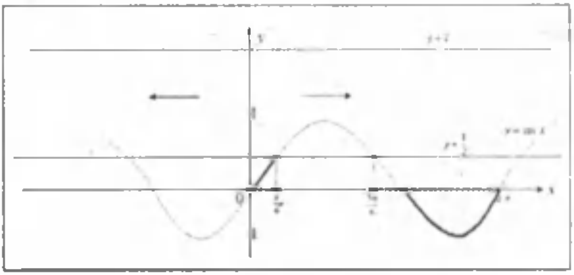
$2a^2-5a+2>0$

$a<\frac{1}{2}$,

$a>2$

Endi ushbu

$\begin{cases} \sin x<\frac{1}{2} \\ \sin x>2 \\ 0\le x\le 2\pi \end{cases}$



sistemaga ega bo'lib, uni grafik yordamida yechamiz. Bunda $\sin x>2$ tengsizlikning yechimi Ø dan iborat.

$x\in\left[0;\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6};2\pi\right]$ **Javob: C.**

26. (05-116-34). $(\pi-e)^{\ln(-\cos^4 x+\sin^4 x)}\ge 1$ tengsizlikning $[0;\pi]$ oraliqqa tegishli barcha yechimlarini aniqlang.

A) $\left(\frac{\pi}{4};\frac{3\pi}{4}\right)$ B) $\left[0;\frac{\pi}{4}\right)\cup\left(\frac{3\pi}{4};\pi\right]$ C) $\left[0;\frac{\pi}{2}\right]$

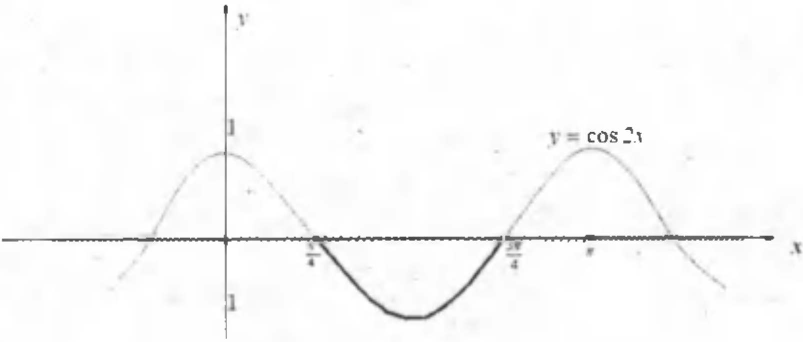
D) $\left[\frac{\pi}{4};\frac{\pi}{2}\right)$ E) $\left[0;\frac{\pi}{2}\right)$

Yechilishi: $0 < \pi - e < 1$ hisobga olib yechamiz

$$\begin{cases} (\pi - e)^{\ln(-\cos^4 x + \sin^4 x)} \geq (\pi - e)^0 \\ 0 \leq x \leq \pi \\ -\cos^4 x + \sin^4 x > 0 \end{cases} \Leftrightarrow \begin{cases} \ln(-\cos^4 x + \sin^4 x) \leq 0 \\ 0 \leq x \leq \pi \\ \cos^4 x - \sin^4 x < 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \cos 2x \geq -1 \\ \cos 2x < 0 \\ 0 \leq x \leq \pi \end{cases} \Leftrightarrow \begin{cases} x \in (-\infty; +\infty) \\ \cos 2x < 0 \\ 0 \leq x \leq \pi \end{cases}$$

Endi grafik yordamida oxirgi sistemaning yechimini osonlik bilan olish mumkin.



$x \in \left(\dfrac{\pi}{4}; \dfrac{3\pi}{4} \right)$. **Javob: A.**

27. (07-103-24). $\cos^2 \dfrac{x}{4} > \dfrac{\sqrt{2}}{2} + \sin^2 \dfrac{x}{4}$ tengsizlikni yeching

A) $\dfrac{\pi}{8} + \pi n < x < \dfrac{7\pi}{8} + \pi n, n \in Z$

B) $\dfrac{\pi}{8} + 2\pi n < x < \dfrac{7\pi}{8} + 2\pi n, n \in Z$

C) $\frac{\pi}{4}+2\pi n<x<\frac{7\pi}{4}+2\pi n, n\in Z$

D) $-\frac{\pi}{2}+4\pi n<x<\frac{\pi}{2}+4\pi n, n\in Z$

Yechilishi:

$$\cos^2\frac{x}{4}-\sin^2\frac{x}{4}>\frac{\sqrt{2}}{2}$$

$$\cos\left(2\cdot\frac{x}{4}\right)>\frac{\sqrt{2}}{2}$$

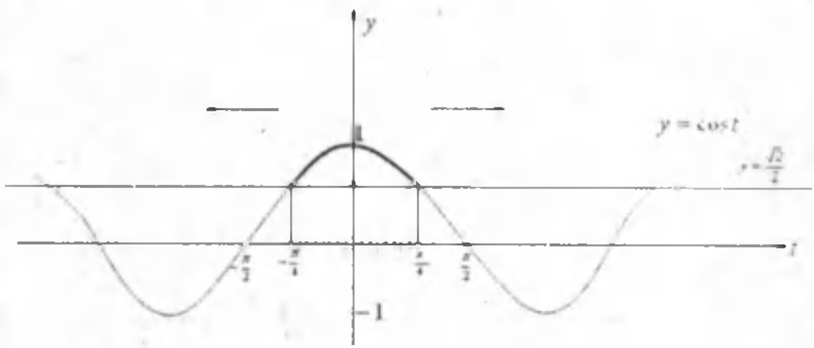
$$\cos\frac{x}{2}>\frac{\sqrt{2}}{2}$$

Oldin tengsizlikni $t=\frac{x}{2}$ deb belgilaymiz

$$-\frac{\pi}{4}+2\pi n<t<\frac{\pi}{4}+2\pi n$$

$$-\frac{\pi}{4}+2\pi n<\frac{x}{2}<\frac{\pi}{4}+2\pi n$$

$$-\frac{\pi}{2}+4\pi n<x<\frac{\pi}{2}+4\pi n,\ n\in Z$$



Javob: D.

28. (07-105-24). $\sin x\cos x<\frac{\sqrt{2}}{4}$ tengsizlikni yeching.

A) $-\frac{\pi}{4}+\pi k<x<\frac{3\pi}{4}+\pi k, k\in Z$

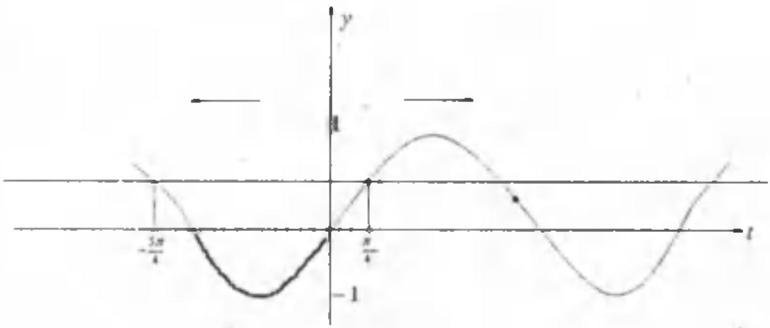
B) $-\frac{5\pi}{8}+\pi k<x<\frac{\pi}{8}+\pi k, k\in Z$

C) $\frac{\pi}{8}+\pi k\le x\le\frac{3\pi}{8}+\pi k, k\in Z$

D) $\frac{\pi}{8}+\pi k<x<\frac{3\pi}{8}+\pi k, k\in Z$

Yechilishi: $\sin x\cos x<\frac{\sqrt{2}}{4}$ $/*2$ $\Rightarrow$ $\sin 2x<\frac{\sqrt{2}}{2}$

$2x=t$ deb belgilaymiz va grafikdan foydalanib, uni t o'zgaruvchiga nisbatan yechib olamiz.



$$-\frac{5\pi}{4}+2\pi n<t<\frac{\pi}{4}+2\pi n \Rightarrow -\frac{5\pi}{4}+2\pi n<2x<\frac{\pi}{4}+2\pi n \Rightarrow -\frac{5\pi}{8}+\pi n<x<\frac{\pi}{8}+\pi n. n\in Z$$

Javob: B.

29. (07-148-24). $\cos 2x\ge-\frac{1}{2}$ tengsizlikning $[0;1,5\pi]$ kesmadagi yechimini toping.

A) $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3};\frac{4\pi}{3}\right]$ B) $\left[\frac{\pi}{3};\frac{2\pi}{3}\right]$ C) $\left[\frac{4\pi}{3};2\pi\right]$

D) $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3};\pi\right]$

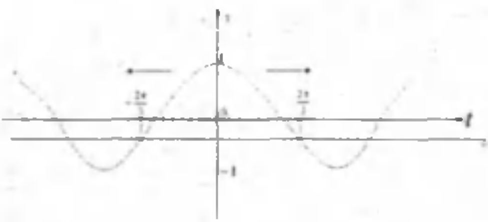
Yechilishi: 2x=t deb belgilaymiz.

$$\cos t \geq -\frac{1}{2}$$

$$-\frac{2\pi}{3}+2\pi n\leq t\leq\frac{2\pi}{3}+2\pi n$$

$$-\frac{2\pi}{3}+2\pi n\leq 2x\leq\frac{2\pi}{3}+2\pi n$$

$$-\frac{\pi}{3}+\pi n\leq x\leq\frac{\pi}{3}+\pi n,\ n\in Z$$



1) n=0 da

$$\begin{cases}-\frac{\pi}{3}\leq x\leq\frac{\pi}{3}\\ 0\leq x\leq\frac{3\pi}{2}\end{cases}\Rightarrow x\in\left[0;\frac{\pi}{3}\right] \qquad (A)$$

2) n=1 da

$$\begin{cases}\frac{2\pi}{3}\leq x\leq\frac{4\pi}{3}\\ 0\leq x\leq\frac{3\pi}{2}\end{cases}\Rightarrow x\in\left[\frac{2\pi}{3};\frac{4\pi}{3}\right] \qquad (B)$$

Endi masalaning yechimi $(A)\cup(B)$ dan, ya'ni $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3};\frac{4\pi}{3}\right]$ dan iborat bo'ladi.

Javob: A.

MUNDARIJA

Izoh va qaydlar uchun

Izoh va qaydlar uchun

Izoh va qaydlar uchun

Ilmiy-ommabop nashr

Umid ISMOILOV
Hamid BOBOJONOV

TRIGONOMETRIYADAN MASALALAR YECHISH

Oliy o'quv yurtlariga kiruvchilar uchun metodik qo'llanma

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Musahhih: Shahnoza To'raxo'jayeva
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Nashriyot raqami № 001

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