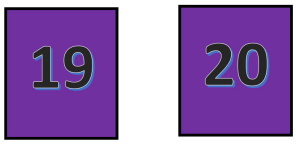
TASHKENT INTERNATIONAL MATHEMATICS OLYMPIAD

3.1 ballga baholanadigan savollar.

1. **(3.1 ball.)** Shodibekda har birida sonlar yozilgan quyidagi ikkita kartochkalar mavjud. Shodibek ulardan foydalanib buyuk matematik olimning tug‘ilgan hamda vafot etgan yillarini hosil qila olishini aniqladi. Ushbu matematik olim necha yil umr ko‘rganini aniqlang.



A) 99 *B*) 100 *C*) 97 *D*) 101

2. **(3.1 ball.)** Hisoblang: $\frac{1}{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}} - \frac{\sqrt[3]{3}}{5} = ?$

A) $\frac{\sqrt[3]{3}}{5}$ *B*) 0 *C*) $\frac{\sqrt[3]{2}}{5}$ *D*) $\frac{\sqrt[3]{2}+1}{5}$

3. **(3.1 ball.)** Tenglamani yeching: $\sqrt[3]{x^x} = x^{\sqrt[3]{x^2}}$.

A) 1;27 *B*) 3;27 *C*) 9;27 *D*) 1;3;9;27

4. **(3.1 ball.)** Teng yonli ACB uchburchakda A va C burchaklar teng. $AB : AC = 5 : 3$ bo‘lsa, $AB - AC = 3$ ga teng. Uchburchak perimetrini toping.

A) 19.5 *B*) 18.5 *C*) 17.5 *D*) 16

5. **(3.1 ball.)** Kitob sahifalarini raqamlash uchun kitob sahifalari sonidan 2 marta ko‘p raqam ishlatildi. Agar kitob sahifalari 500 tadan kam bo‘lsa, sahifalar sonini toping.

A) 108 *B*) 111 *C*) 152 *D*) 153

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4.2 ballga baholanadigan savollar.

6. **(4.2 ball.)** $f(x) = \left[\frac{x}{15}\right] \cdot \left[-\frac{15}{x}\right]$ funksiya $\forall x \in (0; 90)$ da funksiyaning qiymatlar sohasidagi elementlar sonini toping?

A) 5 *B*) 6 *C*) 7 *D*) cheksiz ko‘p

7. **(4.2 ball.)** $y = \frac{6.86}{3.14}x + \frac{6.86}{3.14}$ va $y = \frac{3.14}{6.86}x + \frac{3.14}{6.86}$ to‘g‘ri chiziqlar (x, y) nuqtada kesishadi. U holda $y - x$ ning qiymatini toping.

A) 2.72 *B*) 3.43 *C*) 2 *D*) 1

8. **(4.2 ball.)** ABC uchburchakka ichki chizilgan aylana AB va AC tomonlarga M va N nuqtalarda urinadi. Agar $AB = 4$, $AC = 3$, $BC = 2$ bo‘lsa, AMN uchburchak yuzini toping.

A) $\frac{25\sqrt{15}}{64}$ *B*) $\frac{16\sqrt{15}}{27}$ *C*) $\frac{15\sqrt{15}}{16}$ *D*) $\frac{25\sqrt{15}}{32}$

9. **(4.2 ball.)** x haqiqiy son uchun ushbu ifodaning eng katta qiymatini toping:

$$\frac{120}{|x+7| + |x+4| + |x-5|}.$$

A) 6 *B*) 7 *C*) 8 *D*) 10

10. **(4.2 ball.)** x, y natural sonlar uchun quyidagi tenglik o‘rinli:

$$\sqrt{x - \sqrt{y}} + \sqrt{x + \sqrt{y}} = \sqrt{xy}.$$

U holda $x^2 + y^2$ ning qiymatini toping.

A) 25 *B*) 13 *C*) 34 *D*) 17

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5.3 ballga baholanadigan savollar.

11. **(5.3 ball.)** Agar $a^2-a-1=0$ tenglik o‘rinli bo‘lsa, a^5-5a ning qiymatini toping?

$A)$ 3 $B)$ 5 $C)$ 7 $D)$ 6

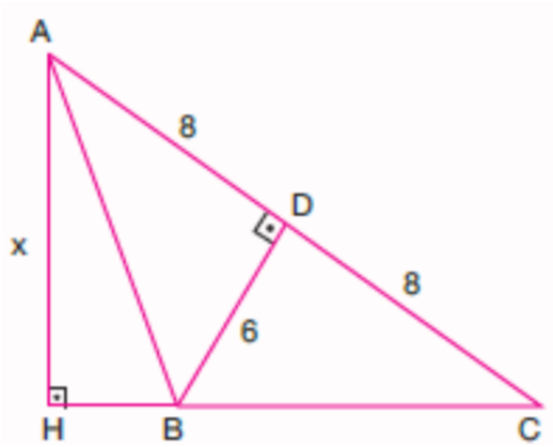
12. **(5.3 ball.)** $a,b,c\in\mathbb{R}$ sonlarda quyidagi berilgan

$$a^2+b^2+c^2+(a+b+c)^2=27$$

bo‘lsa $(a+b)(b+c)(c+a)$ ning eng katta qiymatini toping?

$A)$ 18 $B)$ 30 $C)$ 27 $D)$ 36

13. **(5.3 ball.)** Quyidagi chizmaga ko‘ra $BD=6$, $AD=8$, $CD=8$ va BD kesma AC kesmaga perpendikulyar bo‘lsa AH kesmaning uzunligini toping.



$A)$ 9.6 $B)$ 10 $C)$ 12.4 $D)$ 12

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14. **(5.3 ball.)** a, b, c, x, y, z musbat sonlar uchun $cy+bz = a$, $az+cx = b$ va $bx+ay = c$ o‘rinli bo‘lsin. U holda ushbu ifodaning eng kichik qiymatini toping:

$$\frac{x^2}{1+x} + \frac{y^2}{1+y} + \frac{z^2}{1+z}$$

A) $\frac{\sqrt{3}}{2}$ *B)* $\frac{2}{3}$ *C)* $\frac{\sqrt{2}}{3}$ *D)* $\frac{1}{2}$

15. **(5.3 ball.)** Ushbu to‘plamlarni olaylik $M = \{(x, y) \mid x^2 + 2y^2 = 3\}$ va $N = \{(x, y) \mid y = mx + b\}$. Agar ixtiyoriy $m \in \mathbb{R}$ uchun $M \cap N \neq \emptyset$ bo‘lsa, u holda b ning qabul qilishi mumkin bo‘lgan oralig‘ini belgilang.

A) $\left[-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}\right]$ *B)* $\left(-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}\right)$ *C)* $\left(-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}\right]$ *D)* $\left[-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right]$

7.4 ballga baholanadigan savollar.

16. **(7.4 ball.)** Quyidagi tenglik berilgan:

$$\frac{((3!)!)!}{3!} = k \cdot n!$$

bu yerda k va n musbat butun sonlar hamda n qabul qilishi mumkin bo‘lgan eng katta qiymatda. U holda $k + n$ ning qiymatini toping?

Javob:

17. **(7.4 ball.)** Perimetri 2025 ga teng va tomonlari uzunliklari butun son bo‘lgan nechta uchburchak bor

Javob:

18. **(7.4 ball.)** a, b natural sonlar (turli bo‘lishi shart emas) $\{1, 2, \cdots, 2025\}$ to‘plamdan tavakkaliga bir-biriga bog‘liq bo‘lmagan holda tanlanadi. Quyidagi

$$\lfloor\sqrt{a}\rfloor + \lceil\sqrt{b}\rceil = \lceil\sqrt{a}\rceil + \lfloor\sqrt{b}\rfloor$$

tenglikning o‘rinli bo‘lish ehtimolligi $\frac{m}{n}$ ga teng bo‘lsin, bunda m va n o‘zaro tub natural sonlar. U holda $m + n$ ning qiymatini toping. Bu yerda haqiqiy x son uchun, $\lfloor x\rfloor$ bilan x dan kichik yoki teng bo‘lgan eng katta butun sonni hamda $\lceil x\rceil$ bilan x dan katta yoki teng bo‘lgan eng kichik butun sonni belgilaymiz.

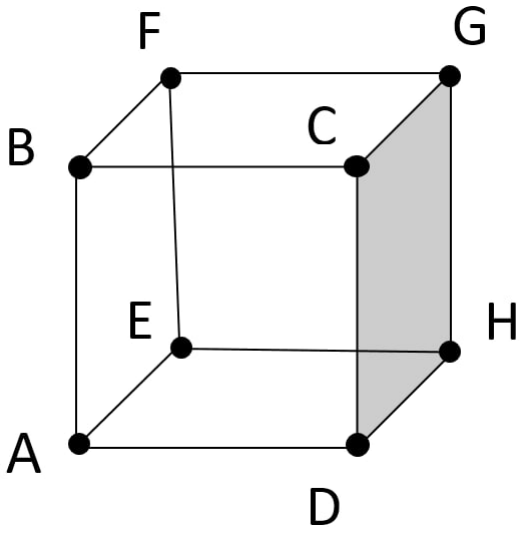
Javob:

19. **(7.4 ball.)** Hisoblang:

$$\frac{1^{-6}+3^{-6}+5^{-6}+7^{-6}+...}{2^{-6}+4^{-6}+6^{-6}+8^{-6}+...}$$

Javob:

20. **(7.4 ball.)** Qirrasi 1 ga teng bo‘lgan $ABCDEFGH$ kub berilgan bo‘lsin. Bu kubni AB qirrasiga nisbatan soat mili bo‘yicha 120° ga burganda yangi $A'B'C'D'E'F'G'H'$ kub hosil bo‘ldi, bu yerda $A = A'$ va $B = B'$. Agar $EGG'D'$ siniq chiziqning uzunligi $\sqrt{m}+\sqrt{n}+\sqrt{k}$ ga teng bo‘lsa, u holda $m+n+k$ ning qiymatini toping. Bu yerda m, n, k natural sonlar.



Javob: